Boundary behavior of analytic functions in two variables via operator theory

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Abstract: A Hilbert space model is an operator theoretic construct that allows the investigation of the behavior of analytic functions in two or more variables by way of Hilbert space geometry. In particular, every analytic map \( \varphi \) of the complex unit bidisk \( \mathbb{D}^2 \) into the complex unit disk \( \mathbb{D} \) has an associated Hilbert space model. Such functions are called Schur-Agler functions.

One important application of Hilbert space models is the generalization of classical theorems of complex analysis. For one variable analytic functions \( f : \mathbb{D} \to \mathbb{D} \), the Carathéodory-Julia Theorem characterizes the behavior of \( f \) at a boundary point \( \tau \in \mathbb{T} \). In particular, the theorem asserts that at \( \tau \) for which \( f \) satisfies a growth condition on regions that approach \( \tau \) non-tangentially, \( f \) is differentiable.

Singularities in functions \( \varphi \) of two complex variables are significantly more complicated than in one variable, even for rational functions. We will discuss the generalization of the Carathéodory-Julia theorem to two variables, and the existence and structure of derivatives to Schur-Agler functions at boundary singularities that meet an analogue of the classical growth condition. We will also characterize the differential structure of \( \varphi \) in terms of a positive contraction operator.

Further Information
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