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The present universe in the Einstein frame, metric-affine $R + 1/R$ gravity

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Abstract

We study the present, flat isotropic universe in $1/R$-modified gravity. We use the Palatini (metric-affine) variational principle and the Einstein (metric-compatible connected) conformal frame. We show that the energy density scaling deviates from the usual scaling for nonrelativistic matter, and the largest deviation occurs in the present epoch. We find that the current deceleration parameter derived from the apparent matter density parameter is consistent with observations. There is also a small overlap between the predicted and observed values for the redshift derivative of the deceleration parameter. The predicted redshift of the deceleration-to-acceleration transition agrees with that in the ΛCDM model but it is larger than the value estimated from SNIa observations.

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1. Introduction

The most accepted explanation of current cosmic acceleration [1–3] is that the universe is dominated by dark energy [4]. However, it is also possible to modify Einstein's general relativity to obtain gravitational field equations that allow accelerated expansion. A particular class of alternative theories of gravity that has recently attracted a lot of interest is that of the $f(R)$ gravity models, in which the gravitational Lagrangian is a function of the curvature scalar R [5]. It has been shown that cosmic acceleration may originate from the addition of a term R^{-1} (or other negative powers of R) to the Einstein–Hilbert Lagrangian R [6–11].

As in general relativity, these models obtain the field equations by varying the total action for both the field and matter. In this paper, we use the metric-affine (Palatini) variational principle, according to which the metric and connection are considered as geometrically independent quantities, and the action is varied with respect to both of them [12]. The other one is the metric (Einstein–Hilbert) variational principle, according to which the action is varied with respect to the metric tensor $g_{\mu\nu}$, and the affine connection coefficients are

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the Christoffel symbols of $g_{\mu\nu}$. Both approaches give the same result only if we use the standard Einstein–Hilbert action. The field equations in the Palatini formalism are second-order differential equations, while for metric theories they are fourth-order [13]. Another remarkable property of the metric-affine approach is that the field equations in vacuum reduce to the standard Einstein equations of general relativity with a cosmological constant [14].

One can show that any of these theories of gravitation is conformally equivalent to the Einstein theory of the gravitational field interacting with additional matter fields [15]. This can be done by means of a Legendre transformation, which in classical mechanics replaces the Lagrangian of a mechanical system with the Helmholtz Lagrangian [16]. For an $f(R)$ gravity, the scalar degree of freedom due to the occurrence of nonlinear second-order terms in the Lagrangian is transformed into an auxiliary scalar field ϕ. The set of variables $(g_{\mu\nu}, \phi)$ is commonly called the *Jordan conformal frame*.

In the Jordan frame, the connection is not metric compatible. The compatibility can be restored by a certain conformal transformation of the metric: $g_{\mu\nu} \to h_{\mu\nu} = f'(R)g_{\mu\nu}$. The new set $(h_{\mu\nu}, \phi)$ is called the *Einstein conformal frame*. Although both frames are equivalent mathematically, they are *not* equivalent physically [17], and the interpretation of cosmological observations can drastically change depending on the adopted frame [18]. In the Palatini formalism, which frame is physical is a matter of choice, although this question should be ultimately answered by experiment or observation.

The consistence of metric-affine $f(R)$ gravities with cosmological observations has already been studied for the Jordan frame [19]. In this paper, we regard the Einstein conformal frame as the physical one. We assume that the gravitational Lagrangian contains the usual Einstein–Hilbert part R and the $1/R$ term which causes current cosmic acceleration [7]. In section 2, we review the metric-affine formalism for an $f(R)$ gravity in the Einstein frame [20]. In section 3, we apply the gravitational field equations for the $R + 1/R$ Lagrangian to a homogeneous and isotropic universe, and solve them for the early and late universe in the quadratic approximation. In section 4, we derive the relations between the present deceleration parameter q_0, the matter density parameter Ω_M and the redshift of the deceleration-to-acceleration transition z_t in the Einstein frame of the metric-affine $1/R$-modified gravity. We also compare these relations with cosmological observations. The results are summarized in section 5.

2. Metric-affine formalism in the Einstein conformal frame

The action for an $f(R)$ gravity in the original (Jordan) frame with the metric $\tilde{g}_{\mu\nu}$ is given by

$$S_J = -\frac{1}{2\kappa c}\int \mathrm{d}^4x[\sqrt{-\tilde{g}}L(\tilde{R})] + S_m(\tilde{g}_{\mu\nu}, \psi). \tag{1}$$

Here, $\sqrt{-\tilde{g}}L(\tilde{R})$ is a Lagrangian density that depends on the curvature scalar $\tilde{R} = R_{\mu\nu}(\Gamma^{\,\lambda}_{\rho\sigma})\tilde{g}^{\mu\nu}$, S_m is the action for matter represented symbolically by ψ and independent of the connection, and $\kappa = \frac{8\pi G}{c^4}$. Tildes indicate quantities calculated in the Jordan frame.

Variation of the action S_J with respect to $\tilde{g}_{\mu\nu}$ yields the field equations

$$L'(\tilde{R})R_{\mu\nu} - \tfrac{1}{2}L(\tilde{R})\tilde{g}_{\mu\nu} = \kappa T_{\mu\nu}, \tag{2}$$

where the dynamical energy–momentum tensor of matter is generated by the Jordan metric tensor:

$$\delta S_m = \frac{1}{2c}\int \mathrm{d}^4x\sqrt{-\tilde{g}}T_{\mu\nu}\delta\tilde{g}^{\mu\nu}, \tag{3}$$

and the prime denotes the derivative of a function with respect to its variable. From variation of S_J with respect to the connection $\Gamma^{\,\rho}_{\mu\,\nu}$, it follows that this connection is given by the Christoffel symbols of the conformally transformed metric [20]:

$$g_{\mu\nu} = L'(\tilde{R})\tilde{g}_{\mu\nu}. \tag{4}$$

The metric $g_{\mu\nu}$ defines the Einstein frame, in which the connection is metric compatible.

The action (1) is dynamically equivalent to the following Helmholtz action [13, 16]:

$$S_H = -\frac{1}{2\kappa c}\int \mathrm{d}^4x\sqrt{-\tilde{g}}[L(\phi(p)) + p(\tilde{R} - \phi(p))] + S_m(\tilde{g}_{\mu\nu}, \psi), \tag{5}$$

where p is a new scalar variable. The function $\phi(p)$ is determined by

$$\left.\frac{\partial L(\tilde{R})}{\partial \tilde{R}}\right|_{\tilde{R}=\phi(p)} = p. \tag{6}$$

From equations (4) and (6), it follows that

$$\phi = RL'(\phi). \tag{7}$$

In the Einstein frame, the action (5) becomes the standard Einstein–Hilbert action of general relativity with additional scalar field:

$$S_E = -\frac{1}{2\kappa c}\int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{\phi(p)}{p} + \frac{L(\phi(p))}{p^2}\right] + S_m(p^{-1}g_{\mu\nu}, \psi). \tag{8}$$

Here, R is the curvature scalar of the metric $g_{\mu\nu}$. Choosing ϕ as the scalar variable leads to [20]

$$S_E = -\frac{1}{2\kappa c}\int \mathrm{d}^4x\sqrt{-g}\,[R - 2V(\phi)] + S_m([L'(\phi)]^{-1}g_{\mu\nu}, \psi), \tag{9}$$

where $V(\phi)$ is the effective potential,

$$V(\phi) = \frac{\phi L'(\phi) - L(\phi)}{2[L'(\phi)]^2}. \tag{10}$$

Variation of the action (9) with respect to $g_{\mu\nu}$ yields the equation of the gravitational field in the Einstein frame [20, 21]:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{\kappa T_{\mu\nu}}{L'(\phi)} - V(\phi)g_{\mu\nu}, \tag{11}$$

while variation with respect to ϕ reproduces equation (7). Contracting (11) with the metric tensor $g^{\mu\nu}$ gives

$$R = -\frac{\kappa T}{L'(\phi)} + 4V(\phi), \tag{12}$$

which is an equation for R since both ϕ and $T = T_{\mu\nu}g^{\mu\nu}$ depend only on R due to (7) and (12). This is equivalent to [20]

$$\phi L'(\phi) - 2L(\phi) = \kappa T L'(\phi). \tag{13}$$

Let us consider the CDTT Lagrangian [7, 8]

$$L(\phi) = \phi - \frac{\alpha^2}{3\phi}, \tag{14}$$

where α is a positive constant[1,2]. Equation (13) for $T = 0$ yields one de Sitter solution,

$$\phi_{\mathrm{ca}} = -\alpha, \tag{15}$$

[1] Equation (6) states that ϕ is the curvature scalar in the Jordan frame $\tilde{R}$. The Lagrangian (14) can thus be written as $L(\tilde{R}) = \tilde{R} - \frac{\alpha^2}{3\tilde{R}}$ and such a model is referred to as the $R + 1/R$ gravity or $1/R$-modified gravity.

[2] The Lagrangian of [20] has an additional term R^3, which is the simplest way of introducing inflation in a metric-affine $f(R)$ gravity [22]. In this work, we omit this term since we are interested in later epochs.

describing present cosmic acceleration [8]. The corresponding value of the curvature scalar in the Einstein frame is determined by equation (7):

$$R = \frac{\phi}{1 + \frac{\alpha^2}{3\phi^2}}, \tag{16}$$

from which we obtain

$$R_{\mathrm{ca}} = -\frac{3\alpha}{4}. \tag{17}$$

Finally, the Einstein equations (11) become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{\kappa T_{\mu\nu}}{1 + \frac{\alpha^2}{3\phi^2}} - \frac{\frac{\alpha^2}{3\phi}}{\left(1 + \frac{\alpha^2}{3\phi^2}\right)^2} g_{\mu\nu}. \tag{18}$$

Equations (16) and (18) give the relation between matter and geometry in the Einstein frame of $1/R$-modified gravity[3].

3. The field equations in the early and late universe

We begin with the Einstein equations for the early universe in which the $1/R$ term does not dominate. In such a universe $\kappa T \gg \alpha$ and we can study the field equations in the second approximation of a small quantity $\frac{\alpha}{\kappa T}$. Equation (13) becomes cubic in ϕ:

$$\phi^3 + \kappa T\phi^2 - \alpha^2\phi + \frac{\alpha^2\kappa T}{3} = 0, \tag{19}$$

and its solution which deviates from $-\kappa T$ (the solution for $\alpha = 0$) by terms linear and quadratic in $\frac{\alpha}{\kappa T}$ is

$$\phi = -\kappa T - \frac{4\alpha^2}{3\kappa T} + O(\alpha^3). \tag{20}$$

In this approximation, the Einstein equations become

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \kappa_{\mathrm{eff}}T_{\mu\nu} + \Lambda_{\mathrm{eff}}g_{\mu\nu}, \tag{21}$$

where

$$\kappa_{\mathrm{eff}} = \kappa\left(1 - \frac{\alpha^2}{3\kappa^2T^2}\right) \tag{22}$$

is the effective gravitational coupling constant and

$$\Lambda_{\mathrm{eff}} = \frac{\alpha^2}{3\kappa T} \tag{23}$$

is the effective cosmological constant.

We now proceed to the field equations in the early Friedmann–Lemaître–Robertson–Walker (FLRW) universe. Let us consider a homogeneous and isotropic universe which is spatially flat [3]. In this case, the interval is given by

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)(\mathrm{d}t^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \tag{24}$$

where $a(t)$ is the scale factor. Moreover, the energy–momentum tensor of matter in the comoving frame of reference is diagonal:

$$T_\mu^\nu = \mathrm{diag}(\epsilon, -P, -P, -P), \tag{25}$$

[3] The value of α is on the order of 10^{-53} m^{-2} [8].

where ϵ is the energy density and P denotes pressure. For this metric and for the case of dust ($P=0$), equation (21) has two independent components:

$$\frac{3\dot{a}^2}{c^2a^2} = \kappa\Im, \tag{26}$$

$$\frac{\dot{a}^2+2a\ddot{a}}{c^2a^2} = \kappa\wp, \tag{27}$$

where the generalized energy density $\Im$ in this approximation is equal to the energy density ϵ and the generalized pressure is given by

$$\wp = -\frac{\alpha^2}{3\kappa^2\epsilon}. \tag{28}$$

The dot denotes the differentiation with respect to time.

From the conservation law for the generalized quantities:

$$\dot{\Im} + 3H(\Im+\wp) = 0, \tag{29}$$

where $H=\frac{\dot{a}}{a}$ is the Hubble parameter, we obtain the scaling law for the energy density:

$$\epsilon^2 = \frac{E_0^2}{a^6} - \frac{\alpha^2}{3\kappa^2}, \tag{30}$$

where E_0 is a positive constant proportional to the total energy of matter within a sphere of radius a in the limit $\frac{\alpha}{\kappa\epsilon}\to 0$. In the course of time, a increases but the right-hand side of (30) never becomes negative since the assumption $\kappa T \gg \alpha$ ceases to hold when $a^3 \sim \frac{E_0\kappa}{\alpha}$. For $b(t) = a^{3/2}(t)$, we obtain

$$\frac{16}{3c^4}(\dot{b})^4 = 3\kappa^2E_0^2 - \alpha^2b^4. \tag{31}$$

For a very late universe $\kappa T \ll \alpha$, and the Einstein equations in the second approximation of a small quantity $\frac{\kappa T}{\alpha}$ are [20]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{3}{4}\kappa T_{\mu\nu}\left(1+\frac{\kappa T}{3\alpha}\right) + \Lambda g_{\mu\nu}, \tag{32}$$

where $\Lambda = \frac{3\alpha}{16} - \frac{\kappa^2T^2}{16\alpha}$. The coupling between matter and the gravitational field is decreased by a factor which tends to 3/4 as $T\to 0$. Equation (32) can be rewritten as

$$\frac{R_{\mu\nu} - g_{\mu\nu}\left(\frac{R}{3} - \frac{R^2}{9\alpha} + \frac{\alpha}{8}\right)}{\frac{1}{2} - \frac{R}{3\alpha}} = \kappa T_{\mu\nu}. \tag{33}$$

From the same equation it also follows that the energy–momentum tensor in the Einstein frame is not covariantly conserved[4]:

$$T_{\mu\nu}^{\ \ ;\nu} = \frac{\frac{\kappa}{4\alpha}\left(T_{\mu\nu}T^{;\nu} - \frac{1}{2}TT_{;\mu}\right)}{\frac{3}{4}+\frac{\kappa T}{4\alpha}}, \tag{34}$$

unless $T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu} = 0$.[5] In the Jordan frame such a tensor is always conserved [20, 23], although [11] arrives at the different conclusion.

[4] It might seem that this tensor is not conserved for $\alpha = 0$. In this case, however, $T=0$ since $\kappa T \ll \alpha$.

[5] Covariant differentiation of the general equation (11) gives

$$T_{\mu\nu}^{\ \ ;\nu} = \phi^{,\nu}L''(\phi)\left(\frac{T_{\mu\nu}}{L'(\phi)} + \frac{[2L(\phi)-\phi L'(\phi)]g_{\mu\nu}}{2\kappa[L'(\phi)]^2}\right), \tag{35}$$

which for an isotropic universe yields (29).

The generalized energy density in the late FLRW universe with the metric (24) is given by

$$\Im = \frac{3\epsilon}{4} + \frac{3\alpha}{16\kappa} + \frac{3\kappa\epsilon^2}{16\alpha}, \tag{36}$$

and the generalized pressure is

$$\wp = -\frac{3\alpha}{16\kappa} + \frac{\kappa\epsilon^2}{16\alpha}. \tag{37}$$

From the conservation law (29) [20] we obtain the scaling law for the energy density:

$$\epsilon a^3 = E_\infty \, \mathrm{e}^{-\frac{\kappa\epsilon}{6\alpha}}, \tag{38}$$

where E_∞ is a positive constant. In the limit of very small ϵ, the above relation simplifies to the usual expression for nonrelativistic matter, $\epsilon a^3 = E_\infty$, where E_∞ has the meaning of a quantity proportional to the total energy of matter within a sphere of radius a.

Similarly, we obtain the energy density dependence on time:

$$\ln\frac{\epsilon}{\epsilon_0} - \frac{11\kappa(\epsilon - \epsilon_0)}{6\alpha} = -\frac{3ct\sqrt{\alpha}}{4}, \tag{39}$$

where time is measured from the instant at which ϵ is equal to a given value ϵ_0. After some mathematical manipulations, we arrive at the following expression for the scaling factor as a function of time in the quadratic approximation:

$$a^3(t) = \frac{E_\infty}{\epsilon_0}\left(\mathrm{e}^{3H_{\mathrm{ca}}t} - \frac{2\kappa\epsilon_0}{\alpha} + \frac{27\kappa^2\epsilon_0^2}{8\alpha^2}\,\mathrm{e}^{-3H_{\mathrm{ca}}t}\right), \tag{40}$$

where $H_{\mathrm{ca}} = c\sqrt{\alpha/16}$ [20]. The late universe approaches a de Sitter spacetime exponentially fast, as in the Jordan frame [8].

We note that the energy density of matter scales like a^{-3} (nonrelativistic matter) in both considered limits, $\kappa\epsilon \ll \alpha$ and $\kappa\epsilon \gg \alpha$. Therefore, there exists the largest deviation from such a scaling in the present epoch when $\kappa\epsilon$ and α are on the same order.

4. The present universe in $R + 1/R$ gravity

From the 00 component of equation (11) we can obtain the Hubble parameter for a flat FLRW universe filled with dust, as a function of ϕ [20]:

$$H(\phi) = \frac{c}{L'(\phi)}\sqrt{\frac{\phi L'(\phi) - 3L(\phi)}{6}}. \tag{41}$$

For the Lagrangian (14), we find

$$H(\phi) = \frac{c\sqrt{-\phi}}{1 + \frac{\alpha^2}{3\phi^2}}\sqrt{\frac{1}{3} - \frac{2\alpha^2}{9\phi^2}}. \tag{42}$$

Similarly, we derive the deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2}$ using the 11 component:

$$q(\phi) = \frac{2\phi L'(\phi) - 3L(\phi)}{\phi L'(\phi) - 3L(\phi)}. \tag{43}$$

For the Lagrangian (14), we obtain

$$q(\phi) = \frac{5\alpha^2 - 3\phi^2}{4\alpha^2 - 6\phi^2}. \tag{44}$$

The time dependence of q is determined by combining equation (44) with the function $\phi = \phi(t)$ which is an integral of [20]:

$$\dot{\phi} = \frac{c\left(-\phi + \frac{\alpha^2}{\phi}\right)\sqrt{\frac{2\alpha^2}{\phi} - 3\phi}}{1 - \frac{2\alpha^4}{3\phi^4} + \frac{7\alpha^2}{3\phi^2}}. \tag{45}$$

We also find the expression for the apparent matter density parameter $\Omega = \frac{\epsilon_0}{\epsilon_c}$:

$$\Omega_M = 2L'(\phi_0)\frac{\phi_0 L'(\phi_0) - 2L(\phi_0)}{\phi_0 L'(\phi_0) - 3L(\phi_0)}, \tag{46}$$

where $\epsilon_c = \frac{3H_0^2}{\kappa c^2}$ is the critical energy density and the subscript 0 refers to the present time. For the Lagrangian (14), this parameter becomes

$$\Omega_M = \left(1 + \frac{\alpha^2}{3\phi_0^2}\right)\frac{3\alpha^2 - 3\phi_0^2}{2\alpha^2 - 3\phi_0^2}. \tag{47}$$

Substituting $\phi_0 = \phi_0(q_0)$ from equation (44) into (47) gives

$$\Omega_M = \frac{q_0^2 - 1}{q_0 - \frac{5}{4}} \tag{48}$$

or

$$q_0 = \frac{1}{2}\left(\Omega_M - \sqrt{\Omega_M^2 - 5\Omega_M + 4}\right). \tag{49}$$

For the early universe $|\phi| \gg \alpha$ and $q \approx \frac{1}{2}$. For the late universe $\phi \to -\alpha$ and $q \to -1$. The transition from the decelerated ($q > 0$) to accelerated ($q < 0$) phase occurs at $q = 0$, which takes place when ϕ equals

$$\phi_t = -\sqrt{\tfrac{5}{3}}\alpha. \tag{50}$$

The Hubble parameter and the energy density of matter at the time of the deceleration-to-acceleration transition are given, respectively, by

$$H_t = c\frac{\sqrt{5}}{6}\sqrt{\sqrt{\frac{5}{3}}\alpha}, \tag{51}$$

$$\epsilon_t = \sqrt{\frac{5}{27}}\frac{\alpha}{\kappa}. \tag{52}$$

We see that the energy density at this time is on the order of $\frac{\alpha}{\kappa}$, i.e. it scales with the largest deviation from nonrelativistic dust.

In the Jordan frame, the energy density scales like a^{-3} which is equivalent to $(1 + z)^3$, where z is the redshift. The energy density in the Einstein frame is related to that in the Jordan frame by the conformal factor $L'(\phi)$. Therefore,

$$\epsilon = \epsilon_0(1 + z)^3\frac{L'(\phi)}{L'(\phi_0)}. \tag{53}$$

Substituting here $\epsilon(\phi)$ from equation (19) gives

$$\frac{\phi L'(\phi) - 2L(\phi)}{\phi_0 L'(\phi_0) - 2L(\phi_0)}\frac{[L'(\phi_0)]^2}{[L'(\phi)]^2} = (1 + z)^3, \tag{54}$$

from which we obtain $\phi = \phi(z, \phi_0)$. In the case of the Lagrangian (14), ϕ is a solution of a quintic equation. From the redshift dependence of ϕ we can derive the relations $H = H(z)$ and $q = q(z)$ using equations (42) and (44), respectively.

The observed value of the present matter density parameter is $\Omega_M = 0.29 \pm^{0.05}_{0.03}$ [24]. Substituting this value into equation (49) gives the prediction for the apparent deceleration parameter:

$$q_0 = -0.67 \pm^{0.06}_{0.03}, \tag{55}$$

which agrees with the observed $q_0 = -0.74 \pm 0.18$ [24].[6] The deceleration-to-acceleration transition happened in the past and the universe expands with a positive acceleration. The corresponding value of ϕ_0 is determined by equation (44):

$$\phi_0 = (-1.05 \pm 0.01)\alpha. \tag{56}$$

From equation (42), we find

$$\alpha = \left(12.5 \pm^{0.4}_{0.7}\right)\frac{H_0^2}{c^2}. \tag{57}$$

The observed value $H_0 = 71 \pm 4$ km (s Mpc)$^{-1}$ [25] leads to

$$\alpha = \left(7.35 \pm^{1.12}_{1.17}\right) \times 10^{-52}\ \mathrm{m}^{-2}. \tag{58}$$

The deceleration-to-acceleration redshift z_t can be obtained from equations (19), (50), (52) and (53). Substituting the numerical value for ϕ_0 from (56) yields

$$z_t = 0.86 \pm^{0.06}_{0.10}. \tag{59}$$

This value overlaps with the ΛCDM value of $z_t = 0.71 \pm^{0.20}_{0.19}$ [7] but disagrees with the observed $z_t = 0.46 \pm 0.13$ [24]. The extraction of z_t from the data appears, however, to be much less robust than the extraction of q_0 [26]. Therefore, we need stronger measurements of z_t to verify if the $R + 1/R$ gravity is a viable explanation of current cosmic acceleration.

The last non-dimensional parameter in our study is q_1, the coefficient in the Taylor expansion of $q(z)$ around $z = 0$: $q(z) = q_0 + q_1 z$. We use $q_1 = \frac{\partial q}{\partial z}\big|_{z=0} = \frac{\partial q/\partial\phi|_{\phi=\phi_0}}{\partial z/\partial\phi|_{\phi=\phi_0}}$ to obtain

$$q_1 = 9\frac{L'(\phi L'^2 - \phi LL'' - LL')(\phi L' - 2L)}{(\phi L' - 3L)^2(4LL'' - L'^2 - \phi L'L'')}\bigg|_{\phi=\phi_0} \tag{60}$$

from equations (43) and (54). The Lagrangian (14) and the current value of ϕ (56) give

$$q_1 = 0.81 \pm^{0.17}_{0.16}, \tag{61}$$

which slightly overlaps with the observed $q_1 = 1.59 \pm 0.63$ [24].

5. Summary

Current cosmic acceleration can be explained by adding a $1/R$ term to the Einstein–Hilbert Lagrangian. We used the metric-affine variational formalism and chose the Einstein conformal frame to be physical. In this frame we derived the field equations for the early and late matter-dominated FLRW universe, in the quadratic approximation with respect to small quantities. We showed that the largest deviation of the energy density scaling ϵ from the usual scaling of nonrelativistic dust occurs in the present epoch. We did not give the exact form of the field

[6] The predictions for non-dimensional cosmological parameters in the $R + 1/R$ gravity are independent of the value of α.

[7] For a flat universe in the ΛCDM model, z_t is a simple function of Ω_M: $z_t = [\frac{2(1-\Omega_M)}{\Omega_M}]^{1/3} - 1$.

equations for any value of ϵ, although it can be done analytically for the $R + 1/R$ Lagrangian. Instead, we derived the expressions for non-dimensional cosmological parameters to verify if the $R + 1/R$ gravity is compatible with observations.

We found that the current deceleration parameter q_0 derived from the apparent matter density parameter Ω_M is consistent with observations. There is also a tiny overlap between the predicted and observed values for the redshift derivative q_1 of the deceleration parameter. The predicted redshift of the deceleration-to-acceleration transition z_t agrees with that in the ΛCDM model but it is larger than the value estimated from SNIa observations. Since the robustness of the z_t measurements is weaker than that of q_0, the question on the viability of the $R + 1/R$ gravity in the Einstein frame remains open.

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