

The cosmic snap parameter in $f(R)$ gravity

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Abstract

We derive the expression for the snap parameter in $f(R)$ gravity. We use the Palatini variational principle to obtain the field equations and regard the Einstein conformal frame as physical. We predict the present-day value of the snap parameter for the particular case $f(R) = R - \text{const}/R$, which is the simplest $f(R)$ model explaining the current acceleration of the universe.

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1. Introduction

$f(R)$ gravity models, in which the gravitational Lagrangian is a function of the curvature scalar R [1, 2], have recently attracted a lot of interest. They explain how current cosmic acceleration originates from the addition of a term R^{-1} to the Einstein–Hilbert Lagrangian R [3]. As in general relativity, $f(R)$ gravity theories obtain the field equations by varying the total action for both the field and matter, and equalling this variation to zero. Here we use the *metric-affine* (Palatini) variational principle, according to which the metric $g_{\mu\nu}$ and the affine connection $\Gamma_{\mu\nu}^\rho$ are considered as geometrically independent quantities, and the action is varied with respect to both of them [4–7]. The standard approach is the *metric* (Einstein–Hilbert) variational principle, according to which the action is varied with respect to the metric tensor, and the affine connection coefficients are the Christoffel symbols of $g_{\mu\nu}$ [8]. Both approaches only give the same result if we use the standard Einstein–Hilbert action [4]. The field equations in the Palatini formalism are second-order differential equations, while for metric theories they are fourth order [9]. Another remarkable property of the metric-affine approach is that the field equations in vacuum reduce to the standard Einstein equations of general relativity with a cosmological constant [9, 10].

One can show that $f(R)$ theories of gravitation are conformally equivalent to the Einstein theory of the gravitational field interacting with additional matter fields, if the action for matter does not depend on the connection [7, 9, 11]. This can be done by means of a Legendre transformation, which replaces an $f(R)$ Lagrangian with a Helmholtz Lagrangian [9, 12]. For $f(R)$ gravity, the scalar degree of freedom due to the occurrence of nonlinear second-order

terms in the Lagrangian is transformed into an auxiliary scalar field ϕ [11]. The set of variables $(g_{\mu\nu}, \phi)$ is commonly called the *Jordan conformal frame*. In the Jordan frame, the connection is metric incompatible unless $f(R) = R$. The compatibility can be restored by a certain conformal transformation of the metric: $g_{\mu\nu} \rightarrow h_{\mu\nu} = f'(R)g_{\mu\nu}$ [13]. The new set $(h_{\mu\nu}, \phi)$ is called the *Einstein conformal frame*, and we will regard the metric in this frame as physical. Although both frames are equivalent mathematically, they are *not* equivalent physically [14], and the interpretation of cosmological observations can drastically change depending on the adopted frame [15].

$f(R)$ gravity models in both the metric and metric-affine formalism have been compared with cosmological observations by several authors [16–20]. The problem of viability of these models at smaller scales, namely their compatibility with the solar system observations, is a subject of recent debate ([21] and references therein). There are also limits on these models arising from particle physics [22], matter instability [23] and violation of the equivalence principle [14, 24]. Current SNIa observations provide the data on the time evolution of the deceleration parameter q in the form of $q = q(z)$, where z is the redshift [25]. The extraction of the information from these data depends, however, on assumed parametrization of $q(z)$ [26]. For small values of z such a dependence can be linear, $q(z) = q_0 + q_1 z$ [25], but its validity should fail at $z \sim 1$. A convenient method to describe models close to Λ CDM is based on the cosmic jerk parameter j , a dimensionless third derivative of the scale factor with respect to the cosmic time [27, 28]. A deceleration-to-acceleration transition occurs for models with a positive value of j_0 and negative q_0 . The flat Λ CDM models have a constant jerk $j = 1$.

Recently, we showed [29] that the predictions for the current value of the jerk parameter for the particular case $f(R) = R - \frac{\alpha^2}{3R}$, which is the simplest way of introducing the cosmological term in $f(R)$ gravity [3, 20], agree with the SNLS SNIa [30] and x-ray galaxy cluster distance [26] data, but do not agree with the SNIa gold sample data [25]. Moreover, the predicted value of the deceleration parameter in this model agrees with all three data sets [29]. Therefore, $f(R)$ models based on the Palatini variational principle and formulated in the Einstein frame, including the case $f(R) = R - \frac{\alpha^2}{3R}$, provide a possible explanation of current cosmic acceleration. More constraints on these models are likely to come from nondimensional parameters containing higher derivatives of the scale factor, such as the snap parameter $s = \frac{\ddot{a}}{aH^4}$ [28].

In this work, we find the general expression for the snap parameter in $f(R)$ gravity, assuming that the universe is flat, homogeneous, isotropic and pressureless. We use the field equations derived from the Palatini variational principle. We assume that matter is minimally coupled to the metric tensor in the Jordan frame, and then transform to the Einstein frame which we consider physical [6] and in which this coupling is non-minimal [5]. Since the question of which frame is physical and in which frame the matter-metric coupling is minimal should be ultimately answered by experiment or observation, the presented model should be treated as phenomenological. Anticipating cosmological measurements, we predict the current value of the snap parameter for the case $f(R) = R - \frac{\alpha^2}{3R}$.

2. The field equations in $f(R)$ gravity

The action for $f(R)$ gravity in the original (Jordan) frame with the metric $\tilde{g}_{\mu\nu}$ is given by [6]

$$S_J = -\frac{1}{2\kappa c} \int d^4x [\sqrt{-\tilde{g}} f(\tilde{R})] + S_m(\tilde{g}_{\mu\nu}, \psi). \quad (1)$$

Here, $\sqrt{-\tilde{g}}f(\tilde{R})$ is a Lagrangian density that depends on the curvature scalar: $\tilde{R} = R_{\mu\nu}(\Gamma^\lambda_{\rho\sigma})\tilde{g}^{\mu\nu}$, S_m is the action for matter represented symbolically by ψ and independent of the connection and $\kappa = \frac{8\pi G}{c^4}$. Tildes indicate quantities calculated in the Jordan frame.

Varying the action S_J with respect to $\tilde{g}_{\mu\nu}$ yields the field equations:

$$f'(\tilde{R})R_{\mu\nu} - \frac{1}{2}f(\tilde{R})\tilde{g}_{\mu\nu} = \kappa T_{\mu\nu}, \tag{2}$$

where the dynamical energy–momentum tensor of matter, $T_{\mu\nu}$, is generated by the Jordan metric tensor [5, 6]:

$$\delta S_m = \frac{1}{2c} \int d^4x \sqrt{-\tilde{g}} T_{\mu\nu} \delta \tilde{g}^{\mu\nu}, \tag{3}$$

and the prime denotes the derivative of a function with respect to its variable. From vanishing of the variation of S_J with respect to the connection $\Gamma^\rho_{\mu\nu}$ it follows that this connection is given by the Christoffel symbols of the conformally transformed metric [11, 13]

$$g_{\mu\nu} = f'(\tilde{R})\tilde{g}_{\mu\nu}. \tag{4}$$

The metric $g_{\mu\nu}$ defines the Einstein frame, in which the connection is metric compatible.

The action (1) is dynamically equivalent to the following Helmholtz action [6, 9, 11]:

$$S_H = -\frac{1}{2\kappa c} \int d^4x \sqrt{-\tilde{g}} [f(\phi(p)) + p(\tilde{R} - \phi(p))] + S_m(\tilde{g}_{\mu\nu}, \psi), \tag{5}$$

where p is a new scalar variable. The function $\phi(p)$ is determined by

$$\left. \frac{\partial f(\tilde{R})}{\partial \tilde{R}} \right|_{\tilde{R}=\phi(p)} = p. \tag{6}$$

From equations (4) and (6) it follows that

$$\phi = Rf'(\phi), \tag{7}$$

where $R = R_{\mu\nu}(\Gamma^\lambda_{\rho\sigma})g^{\mu\nu}$ is the Riemannian curvature scalar of the metric $g_{\mu\nu}$.

In the Einstein frame, the action (5) becomes the standard Einstein–Hilbert action of general relativity with an additional scalar field:

$$S_E = -\frac{1}{2\kappa c} \int d^4x \sqrt{-g} [R - p^{-1}\phi(p) + p^{-2}f(\phi(p))] + S_m(p^{-1}g_{\mu\nu}, \psi). \tag{8}$$

Choosing ϕ (which is equal to the curvature scalar \tilde{R} in the Jordan frame) as the scalar variable leads to

$$S_E = -\frac{1}{2\kappa c} \int d^4x \sqrt{-g} [R - 2V(\phi)] + S_m([f'(\phi)]^{-1}g_{\mu\nu}, \psi), \tag{9}$$

where $V(\phi)$ is the effective potential:

$$V(\phi) = \frac{\phi f'(\phi) - f(\phi)}{2[f'(\phi)]^2}. \tag{10}$$

Varying the action (9) with respect to $g_{\mu\nu}$ yields the equations of the gravitational field in the Einstein frame [5, 6]:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{\kappa T_{\mu\nu}}{f'(\phi)} - V(\phi)g_{\mu\nu}, \tag{11}$$

while the variation with respect to ϕ reproduces (7). Equations (7) and (11) give

$$\phi f'(\phi) - 2f(\phi) = \kappa T f'(\phi), \tag{12}$$

from which we obtain $\phi = \phi(T)$. Substituting ϕ into the field equations (11) leads to a relation between the Ricci tensor and the energy–momentum tensor,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa_r(T)T_{\mu\nu} + \Lambda(T)g_{\mu\nu}, \quad (13)$$

with a running gravitational coupling $\kappa_r(T) = \kappa[f'(\phi(T))]^{-1}$ and a variable cosmological term $\Lambda(T) = -V(\phi(T))$:

$$\Lambda(\phi) = \frac{f(\phi) - \phi f'(\phi)}{2[f'(\phi)]^2}. \quad (14)$$

Such a relation is in general nonlinear and depends on the form of the function $f(R)$.

The Bianchi identity applied to equation (11) gives

$$T_{\mu\nu}{}^{;\nu} = \phi^{;\nu} f''(\phi) \left(\frac{T_{\mu\nu}}{f'(\phi)} + \frac{[2f(\phi) - \phi f'(\phi)]g_{\mu\nu}}{2\kappa[f'(\phi)]^2} \right). \quad (15)$$

This relation means that the energy–momentum tensor in the Einstein frame is not covariantly conserved, unless $f(R) = R$ or $T = 0$ [20]. We can write the field equation (11) as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa(T_{\mu\nu}^m + T_{\mu\nu}^\Lambda), \quad (16)$$

where $T_{\mu\nu}^m = T_{\mu\nu}$. This defines the dark energy–momentum tensor,

$$T_{\mu\nu}^\Lambda = \frac{\Lambda(\phi)}{\kappa} g_{\mu\nu} + \frac{1 - f'(\phi)}{f'(\phi)} T_{\mu\nu}. \quad (17)$$

From equation (16) it follows that matter and dark energy together form a system that has a conserved 4-momentum. Consequently, in the Palatini $f(R)$ gravity formulated in the Einstein frame, matter and dark energy *interact* ([31] and references therein). This interaction may be responsible for the observed large entropy of the universe.

We assumed that matter is minimally coupled to the metric tensor in the Jordan frame. Then we transformed to the Einstein frame, in which this coupling becomes non-minimal, and assumed that this frame is physical, motivated by the fact that the connection is metric compatible in this frame. Such a construction is completely phenomenological. However, if we consider the Einstein frame from the very beginning and define the energy–momentum tensor generated by the metric tensor $g_{\mu\nu}$ as the true energy–momentum tensor for matter (minimal coupling in the Einstein frame), the resulting action, instead of equation (9), is

$$S_E = -\frac{1}{2\kappa c} \int d^4x \sqrt{-g} [R - 2V(\phi)] + S_m(g_{\mu\nu}, \psi). \quad (18)$$

Consequently, the field ϕ does not couple to anything and we arrive at general relativity with the cosmological constant, which is not interesting from a modified gravity perspective [5, 6].

3. The snap parameter in $f(R)$ gravity

The snap parameter in cosmology is defined as [28]

$$s = \frac{\ddot{a}}{aH^4}, \quad (19)$$

where a is the cosmic scale factor, H is the Hubble parameter, and the dot denotes differentiation with respect to the cosmic time. This parameter appears in the fourth-order term of the Taylor expansion of the scale factor around a_0 :

$$\begin{aligned} \frac{a(t)}{a_0} = & 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2(t - t_0)^2 + \frac{1}{6}j_0 H_0^3(t - t_0)^3 \\ & + \frac{1}{24}s_0 H_0^4(t - t_0)^4 + O[(t - t_0)^5], \end{aligned} \quad (20)$$

where the subscript 0 denotes the present-day value. We can rewrite equation (19) as

$$s = \frac{\dot{j}}{H} - j(2 + 3q), \quad (21)$$

where q is the deceleration parameter and j is the jerk parameter. For the flat Λ CDM model $s = -(2+3q)$ since $j = 1$ [26, 29], and the departure of the quantity ds/dq from -3 measures how the evolution of the universe deviates from the Λ CDM dynamics.

From the gravitational field equations (11) applied to a flat Robertson–Walker universe filled with dust we can derive the ϕ -dependence of the Hubble parameter [6]

$$H(\phi) = \frac{c}{f'(\phi)} \sqrt{\frac{\phi f'(\phi) - 3f(\phi)}{6}}, \quad (22)$$

the deceleration parameter [20]

$$q(\phi) = \frac{2\phi f'(\phi) - 3f(\phi)}{\phi f'(\phi) - 3f(\phi)} \quad (23)$$

and the jerk parameter [29]

$$j(\phi) = [2\phi^2 f'^4 + 10\phi^3 f'^3 f'' - 75\phi^2 f'^2 f f'' - 12\phi f f'^3 + 18f^2 f'^2 - 162f^3 f'' + 189\phi f^2 f' f''] \times [(\phi f' - 3f)^2 (2f'^2 + \phi f' f'' - 6f f'')]^{-1}. \quad (24)$$

The prime denotes the differentiation with respect to ϕ . We also have the expression for the time dependence of ϕ [6]:

$$\dot{\phi} = \frac{\sqrt{6c(\phi f' - 2f)} \sqrt{\phi f' - 3f}}{2f'^2 + \phi f' f'' - 6f f''}. \quad (25)$$

For the ϕ -derivative of the jerk parameter we obtain a quite complicated expression:

$$j' = [(\phi f' - 3f)(2f'^2 + \phi f' f'' - 6f f'')(30\phi^3 f'^2 f''^2 + 10\phi^3 f'^3 f''' - 150\phi^2 f f' f''^2 - 37\phi^2 f'^3 f'' - 75\phi^2 f f'^2 f''' - 8\phi f'^4 + 24f f'^3 + 189\phi f^2 f''^2 + 189\phi f^2 f' f''' + 192\phi f f'^2 f'' - 162f^3 f''' - 267f^2 f' f'' - (2\phi^2 f'^4 + 10\phi^3 f'^3 f'' - 75\phi^2 f'^2 f f'' - 12\phi f f'^3 + 18f^2 f'^2 + 189\phi f^2 f' f'' - 162f^3 f'') \times (3\phi^2 f' f''^2 - 15\phi f f''^2 - 8f'^3 + 27f f' f'' - \phi f'^2 f'' + \phi^2 f'^2 f''' - 9\phi f f' f''' + 18f^2 f''')] \times [(\phi f' - 3f)^3 (2f'^2 + \phi f' f'' - 6f f'')]^{-1}. \quad (26)$$

Combining equations (21–25) and using $\dot{j} = \dot{\phi} j'(\phi)$ lead to

$$s = j' \frac{6f'(\phi f' - 2f)}{(2f'^2 + \phi f' f'' - 6f f'')} - j \frac{8\phi f' - 15f}{\phi f' - 3f}. \quad (27)$$

Putting here j from equation (24) and j' from equation (26) gives the final expression for the snap parameter in $f(R)$ gravity as a function of ϕ , $f(\phi)$, $f'(\phi)$, $f''(\phi)$ and $f'''(\phi)$, which we do not write explicitly.

We now examine the case $f(R) = R - \frac{\alpha^2}{3R}$, where α is a constant, which is a possible explanation of current cosmic acceleration [3]. In this model, the present value of ϕ is $\phi_0 = (-1.05 \pm 0.01)\alpha$, where $\alpha = (7.35_{-1.17}^{+1.12}) \times 10^{-52} \text{ m}^{-2}$ [20]. We do not need to know the exact value of α since it does not affect nondimensional cosmological parameters. Substituting ϕ_0 into equations (24), (26) and (27) gives the present value of the cosmic snap parameter¹:

$$s_0 = -0.22_{-0.19}^{+0.21}. \quad (28)$$

¹ The predicted value for the current cosmic jerk parameter found in [29] is $j_0 = 1.01_{-0.21}^{+0.08}$. Here, we recalculated this value and obtained $j_0 = 1.01 \pm 0.01$, which differs from the former by the precision errors. This correction does not change the conclusions of [29].

In the $f(R) = R - \frac{\alpha^2}{3R}$ model, the deceleration-to-acceleration transition occurred at $\phi_t = -\sqrt{5/3}\alpha$ [20]. Consequently, we find the snap parameter at this moment:

$$s_t = -2.68. \quad (29)$$

This value shows that the snap parameter in $f(R)$ gravity changes significantly between the deceleration-to-acceleration transition and now, which is clear from equation (21) and the fact that the deceleration parameter changes in this period of time from 0 to the predicted value $q_0 = -0.67^{+0.06}_{-0.03}$ [20]. For the flat Λ CDM model, the snap parameter increases from $s = -7/2$ for the matter epoch, through $s = -2$ at this transition, to the asymptotic de Sitter value $s = 1$, indicating the difference between the $f(R)$ and Λ CDM predictions for s_t .

Lastly, we show the role of the energy conditions [32] in metric-affine $f(R)$ gravity models. For a pressureless universe, these conditions reduce to the inequality

$$\epsilon \geq 0, \quad (30)$$

where $\epsilon = T$ is the energy density of matter. From equations (12), (22) and (23) we obtain

$$\epsilon = \frac{2H^2(1+q)f'(\phi)}{\kappa c^2}. \quad (31)$$

Since $q > -1$ (it reaches -1 asymptotically [20]) and $f'(\phi) > 0$ (this condition assures that the conformal transformation from the Jordan to the Einstein frame does not change the signature of the metric tensor), formula (30) is satisfied. Therefore, the energy conditions do not impose additional constraints on Palatini $f(R)$ gravity models. For metric $f(R)$ models, these conditions lead to constraints containing the jerk and snap parameter [33]².

4. Summary

We derived the expression for the cosmic snap parameter in $f(R)$ gravity formulated in the Einstein conformal frame. We used the Palatini variational principle to obtain the field equations and apply them to a flat, homogeneous and isotropic universe filled with dust. We considered the particular case $f(R) = R - \frac{\alpha^2}{3R}$, which is the simplest $f(R)$ model explaining the current cosmic acceleration, and for which the predicted present-day values of the deceleration and jerk parameters are quite consistent with cosmological data. For the present-day value of the snap parameter, we predict $s_0 = -0.22^{+0.21}_{-0.19}$.

Expanding the scale factor to fourth order with respect to time (equation (20)) is physically meaningful, since cosmological data already allow us to measure the third-order term: the jerk parameter [26]. The snap parameter may be important for observations involving redshifts $z \sim 1$ and higher, where expansion of cosmological quantities in powers of z cannot be limited only to linear and quadratic terms. Therefore this paper is not only a formal mathematical exercise, but also provides physically measurable constraints on Palatini $f(R)$ gravity.

A cosmological sequence of matter dominance, deceleration-to-acceleration transition and acceleration era may always emerge as cosmological solutions of $f(R)$ gravity [17]. We showed in [31] that, in Palatini $f(R)$ gravity, the deviation of the growth of the cosmic scale factor in the matter era from the standard law $a(t) \sim t^{2/3}$ is small, which is consistent with WMAP cosmological data [34]. On the other hand, metric $f(R)$ gravity models with a power of R dominant at large or small R yield the law $a(t) \sim t^{1/2}$ which is ruled out by cosmological observations [35]. Therefore $f(R)$ gravity in the Palatini variational formalism is a viable theory of gravitation that explains current cosmic acceleration.

² The jerk and snap parameters do not appear in equation (31) and the energy conditions because the field equations are second order, as opposed to the fourth-order field equations in the metric formalism.

It is possible to find an $f(R)$ action without cosmological constant which exactly reproduces the behaviour of the Einstein–Hilbert action with cosmological constant, i.e. the expansion history of the universe does not uniquely determine the form of the gravitational action [18, 19]. Moreover, the background expansion alone cannot distinguish between different choices of $f(R)$ and one must study cosmological perturbations in order to determine which choice is physical [19, 36]. Measurements of higher derivatives of the scale factor, including the snap parameter, will probably constitute a robust part of these studies.

References

- [1] Starobinsky A A 1980 *Phys. Lett. B* **91** 99
Kerner R 1982 *Gen. Rel. Grav.* **14** 453
Barrow J D and Ottewill A C 1983 *J. Phys. A: Math. Gen.* **16** 2757
Duruiseau J P and Kerner R 1986 *Class. Quantum Grav.* **3** 817
- [2] Capozziello S, Cardone V F, Carloni S and Troisi A 2003 *Int. J. Mod. Phys. D* **12** 1969
Allemandi G, Borowiec A, Francaviglia M and Odintsov S D 2005 *Phys. Rev. D* **72** 063505
Carloni S, Dunsby P K S, Capozziello S and Troisi A 2005 *Class. Quantum Grav.* **22** 4839
Nojiri S and Odintsov S D 2007 *Int. J. Geom. Methods Mod. Phys.* **4** 115
Nojiri S and Odintsov S D 2006 *Phys. Rev. D* **74** 086005
- [3] Vollick D N 2003 *Phys. Rev. D* **68** 063510
Nojiri S and Odintsov S D 2003 *Phys. Rev. D* **68** 123512
Carroll S M, Duvvuri V, Trodden M and Turner M S 2004 *Phys. Rev. D* **70** 043528
Meng X H and Wang P 2003 *Class. Quantum Grav.* **20** 4949
Meng X H and Wang P 2004 *Class. Quantum Grav.* **21** 951
Sotiriou T P 2006 *Phys. Rev. D* **73** 063515
Copeland E J, Sami M and Tsujikawa S 2006 *Int. J. Mod. Phys. D* **15** 1753
- [4] Schrödinger E 1954 *Space–Time Structure* (Cambridge: Cambridge University Press)
- [5] Olmo G J and Komp W 2004 *Preprint gr-qc/0403092*
- [6] Popławski N J 2006 *Class. Quantum Grav.* **23** 2011
- [7] Sotiriou T P and Liberati S 2007 *Ann. Phys.* **322** 935
- [8] Landau L D and Lifshitz E M 1975 *The Classical Theory of Fields* (Oxford: Pergamon)
- [9] Magnano G 1995 *Preprint gr-qc/9511027*
- [10] Ferraris M, Francaviglia M and Volovich I 1994 *Class. Quantum Grav.* **11** 1505
- [11] Jakubiec A and Kijowski J 1988 *Phys. Rev. D* **37** 1406
Magnano G and Sokolowski L M 1994 *Phys. Rev. D* **50** 5039
- [12] Ferraris M and Kijowski J 1982 *Gen. Rel. Grav.* **14** 165
- [13] Jakubiec A and Kijowski J 1989 *J. Math. Phys.* **30** 1073
Maeda K-I 1989 *Phys. Rev. D* **39** 3159
- [14] Faraoni V and Gunzig E 1999 *Int. J. Theor. Phys.* **38** 217
- [15] Allemandi G, Capone M, Capozziello S and Francaviglia M 2006 *Gen. Rel. Grav.* **38** 33
- [16] Capozziello S, Cardone V F and Troisi A 2005 *Phys. Rev. D* **71** 043503
Koivisto T 2006 *Phys. Rev. D* **73** 083517
Sotiriou T P 2006 *Class. Quantum Grav.* **23** 1253
Amarzguoui M, Elgarøy Ø, Mota D F and Multamäki T 2006 *Astron. Astrophys.* **454** 707
Brookfield A W, van de Bruck C and Hall L M H 2006 *Phys. Rev. D* **74** 064028
Amendola L, Gannouji R, Polarski D and Tsujikawa S 2007 *Phys. Rev. D* **75** 083504 (*Preprint gr-qc/0612180*)
Fay S, Tavakol R and Tsujikawa S 2007 *Phys. Rev. D* **75** 063509
- [17] Nojiri S and Odintsov S D 2006 *Preprint hep-th/0611071*
- [18] de la Cruz-Dombriz Á and Dobado A 2006 *Phys. Rev. D* **74** 087501
- [19] Multamäki T and Vilja I 2006 *Phys. Rev. D* **73** 024018
- [20] Popławski N J 2006 *Class. Quantum Grav.* **23** 4819
- [21] Faraoni V 2006 *Phys. Rev. D* **74** 023529
Erickcek A L, Smith T L and Kamionkowski M 2006 *Phys. Rev. D* **74** 121501 (R)
Olmo G J 2007 *Phys. Rev. D* **75** 023511
Faraoni V 2007 *Phys. Rev. D* **75** 067302
- [22] Flanagan É É 2004 *Phys. Rev. Lett.* **92** 071101

- [23] Faraoni V 2006 *Phys. Rev. D* **74** 104017
- [24] Olmo G J 2007 *Phys. Rev. Lett.* **98** 061101
- [25] Riess A G *et al* 2004 *Astrophys. J.* **607** 665
- [26] Rapetti D, Allen S W, Amin M A and Blandford R D 2007 *Mon. Not. R. Astron. Soc.* **375** 1510
- [27] Chiba T and Nakamura T 1998 *Prog. Theor. Phys.* **100** 1077
- Sahni V, Saini T D, Starobinsky A A and Alam U 2003 *J. Exp. Theor. Phys. Lett.* **77** 201
- Sahni V 2002 *Preprint astro-ph/0211084*
- Alam U, Sahni V, Saini T D and Starobinsky A A 2003 *Mon. Not. R. Astron. Soc.* **344** 1057
- Blandford R D, Amin M, Baltz E A, Mandel K and Marshall P J 2004 *Preprint astro-ph/0408279*
- [28] Visser M 2004 *Class. Quantum Grav.* **21** 2603
- Visser M 2005 *Gen. Rel. Grav.* **37** 1541
- [29] Popławski N J 2006 *Phys. Lett. B* **640** 135
- [30] Astier P *et al* 2006 *Astron. Astrophys.* **447** 31
- [31] Popławski N J 2006 *Phys. Rev. D* **74** 084032
- [32] Wald R M 1984 *General Relativity* (Chicago: University of Chicago Press)
- [33] Perez Bergliaffa S E 2006 *Phys. Lett. B* **642** 311
- [34] Spergel D N *et al* 2003 *Astrophys. J. Suppl. Ser.* **148** 175
- [35] Amendola L, Polarski D and Tsujikawa S 2007 *Phys. Rev. Lett.* **98** 131302
- [36] Koivisto T and Kurki-Suonio H 2006 *Class. Quantum Grav.* **23** 2355