Big Bounce and Inflation from Spin and Torsion

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Big Bounce

Inflation

Quantum Fluctuations

Dark Ages

Cosmic Microwave Background

Afterglow Light Pattern

380,000 yrs.

1st Stars about 400 million yrs.

Development of Galaxies, Planets, etc.

Dark Energy Accelerated Expansion

Big Bang Expansion

13.7 billion years

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Problems of general relativity

General relativity describes gravity as curvature of spacetime.

• Singularities: points with infinite density of matter.

• Incompatible with quantum mechanics. We need quantum gravity. It may resolve the singularity problem.

• Field equations contain the conservation of orbital angular momentum, contradicting Dirac equation which gives the conservation of total angular momentum (orbital + spin) and allows spin-orbit exchange in QM.

Simplest extension of GR to include QM spin: Einstein-Cartan theory. It also resolves the singularity problem.
Problems of Big-Bang cosmology & inflation

• Big-Bang singularity.

• What caused the Big Bang? What existed before?

• Inflation (exponential expansion of the early Universe) solves the flatness and horizon problems, and predicts the observed spectrum of CMB perturbations. What caused inflation? (hypothetical scalar fields are usually used)

• How did inflation end? (no eternal inflation)

Einstein-Cartan theory replaces the Big Bang by a non-singular **Big Bounce**. The dynamics after the bounce explains the flatness/horizon problems. **NP, PLB 694, 181 (2010).**
Einstein-Cartan-Sciama-Kibble gravity

- Spacetime has curvature and torsion. \[ S^k_{ij} = \Gamma^k_{[ij]} \]

- Lagrangian density is proportional to Ricci scalar (as in GR).

- Cartan equations:
  Torsion is proportional to spin density of fermions. ECSK differs significantly from GR at densities > $10^{45}$ kg/m$^3$; passes all tests.

- Einstein equations:
  Curvature is proportional to energy and momentum density.

arXiv.org > gr-qc > arXiv:0911.0334
Universe with spin fluid

Dirac particles can be averaged macroscopically as a spin fluid.

\[ s_{\mu\nu} = s_{\mu\nu} u^\rho \quad s_{\mu\nu} u_\nu = 0 \quad s^2 = s_{\mu\nu} s_{\mu\nu} / 2 \]

Einstein-Cartan equations for a (closed) FLRW Universe become Friedmann equations for the scale factor \( a \).

\[
\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3} \kappa \left( \epsilon - \frac{1}{4} \kappa s^2 \right) a^2 \\
\frac{\dot{a}^2}{c^2} + 2a\ddot{a} + 1 = -\kappa \left( p - \frac{1}{4} \kappa s^2 \right) a^2 \\
\]

Spin and torsion modify the energy density and pressure with a negative term proportional to the square of the fermion number density \( n \), which acts like repulsive gravity.
Universe with spin fluid

For relativistic matter, Friedmann equations can be written in terms of temperature: \( \varepsilon \approx 3p \sim T^4, n \sim T^3. \)

\[
\frac{\ddot{a}}{c^2} + 1 = \frac{1}{3} \kappa (h_* T^4 - \alpha h_{nf}^2 T^6) a^2 \\
\frac{\dot{a}}{a} + \frac{\dot{T}}{T} = 0 \quad \quad \alpha = \kappa (\hbar c)^2 / 32
\]

Using nondimensional quantities:

\[
x = \frac{T}{T_{cr}} \quad y = \frac{a}{a_{cr}} \quad \tau = \frac{ct}{a_{cr}}
\]

\[
T_{cr} = \left( \frac{2h_*}{3\alpha h_{nf}^2} \right)^{1/2} \quad a_{cr} = \frac{9\hbar c}{8\sqrt{2}} \left( \frac{\alpha h_{nf}^4}{h_*^3} \right)^{1/2}
\]

Generating nonsingular bounce

\[
\dot{y}^2 + 1 = \frac{3C^4}{y^2} - \frac{2C^6}{y^4}
\]

\[
y_{\pm}^2 = 3C^4 \left[ \frac{1 \pm \sqrt{1 - \frac{8}{9C^2}}}{2} \right]
\]

Turning points ($\dot{y} = 0$) for the closed Universe with torsion are positive — **no cosmological singularity!**

- 2 points if $C > (8/9)^{1/2}$ (absolute $y_{\text{min}} = 1$ for $C = 1$)
- 1 point if $C = (8/9)^{1/2}$ -> stationary Universe
- 0 points if $C < (8/9)^{1/2}$ -> Universe cannot exist (form)

<table>
<thead>
<tr>
<th>$C$</th>
<th>$y_{\text{min}}^2$</th>
<th>$y_{\text{max}}^2$</th>
<th>$x_{\text{max}}^2$</th>
<th>$x_{\text{min}}^2$</th>
</tr>
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<tbody>
<tr>
<td>$\sqrt{8/9}$</td>
<td>$\frac{32}{27}$</td>
<td>$\frac{32}{27}$</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{3}{4}$</td>
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<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\gg 1$</td>
<td>$\frac{2C^2}{3}$</td>
<td>$3C^4$</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{1}{3C^2}$</td>
</tr>
</tbody>
</table>

GU & NP (in preparation)

Proposal: the Universe has begun at $a \sim a_{\text{cr}}$ and $T \sim T_{\text{cr}}$ with $C \sim 1$. During inflation, $C$ increased to the current value ($> 10^{30}$).
Generating inflation with only 1 parameter

Near a bounce, particle production enters through a term $\sim H^4$, with $\beta$ as a production parameter.

$$\frac{\dot{a}}{a} \left[ 1 - \frac{3\beta}{c^3 h_n T^3} \left( \frac{\dot{a}}{a} \right)^3 \right] = -\frac{\dot{T}}{T}$$

To avoid eternal inflation: the $\beta$ term $< 1 \rightarrow \beta < \beta_{cr} \approx 1/929$.

For $\beta \approx \beta_{cr}$ and during an expansion phase, when $H = \dot{a}/a$ reaches a maximum, the $\beta$ term is slightly lesser than 1 and:

$$T \sim \text{const}, \quad H \sim \text{const}.$$  

Exponential expansion lasts about $t_{\text{Planck}}$ then $H$ and $T$ decrease. Torsion becomes weak, inflation ends, and radiation dominated era begins. No scalar fields needed.  

Dynamics of the early Universe

$\frac{\beta}{\beta_{cr}} = 0.9998$
$a_0 = 10^{-27}$ m

S. Desai & NP, PLB 755, 183 (2016)
• The numbers of e-folds and bounces (until the Universe reaches the radiation-matter equality) depend on the particle production but are not too sensitive to the initial scale factor.

• The Big Bang was the last bounce (Big Bounce).

<table>
<thead>
<tr>
<th>$\beta/\beta_{cr}$</th>
<th>Number of bounces</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.996</td>
<td>1</td>
</tr>
<tr>
<td>0.984</td>
<td>2</td>
</tr>
<tr>
<td>0.965</td>
<td>3</td>
</tr>
<tr>
<td>0.914</td>
<td>5</td>
</tr>
<tr>
<td>0.757</td>
<td>10</td>
</tr>
</tbody>
</table>

The Universe might have originated from the interior of a black hole. Accordingly, every black hole may create a new, closed, baby universe (Novikov, Pathria, Hawking, Smolin, NP).
It is possible to find a scalar field potential which generates the same time dependence of the scale factor.

\[(\beta/\beta_{cr} = 0.9998)\]

Scalar-field plateau models of inflation: initial conditions problem, eternal inflation, unlikeliness (compared to power-law), several parameters.

Torsion cosmology avoids those problems with only 1 parameter.
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Desai & NP, PLB 755, 183 (2016)
Summary

• The conservation law for total angular momentum (orbital + spin) in curved spacetime, consistent with Dirac equation, requires torsion.

• The simplest theory with torsion, Einstein-Cartan gravity, has the same Lagrangian as GR.

• Torsion is strong only at extremely high densities and manifests itself as gravitational repulsion that avoids the formation of singularities. The Big Bang is replaced by a nonsingular Big Bounce.

• Particle production after a bounce can generate a finite period of inflation which ends when torsion becomes weak. No hypothetical fields or extra dimensions are needed. The dynamics is plateau-like and supported by the Planck data.

• EC gravity is the simplest and most natural explanation of the Big Bounce and inflation.
If quantum effects in the gravitational field near a bounce produce enough matter, then the closed Universe can reach a size at which dark energy becomes dominant and expands to infinity.

Otherwise, the Universe contracts to another bounce (with larger scale factor) at which it produces more matter, and expands again.

(Image: Lord, Tensors, Relativity, and Cosmology.)
From the equivalent scalar field potential, one can calculate the parameters which are being measured in CMB.

Consistent with Planck 2015.

S. Desai & NP, PLB 755, 183 (2016)