

Universe in a black hole with spin and torsion

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Einstein-Cartan gravity

In general relativity (GR), the affine connection Γ_{ij}^k is constrained to be symmetric.

Einstein–Cartan–Sciama–Kibble (EC) gravity removes this constraint by regarding the **antisymmetric part of the connection, the torsion tensor** $S^k_{ij} = \Gamma_{[ij]}^k$, as a variable. The total Lagrangian density is $-\frac{1}{2\kappa}R\sqrt{-g} + L_m$, where R is the Ricci scalar constructed from the connection and L_m is the Lagrangian density of matter, as in GR.

Varying the Lagrangian with respect to the contortion tensor $C_{ijk} = S_{ijk} + S_{jki} + S_{kji}$ gives the Cartan equations:

$$S^j_{ik} - S_i\delta_k^j + S_k\delta_i^j = -\frac{1}{2}\kappa s_{ik}^j,$$

where $S_i = S^k_{ik}$ and $s^{ijk} = 2(\delta L_m/\delta C_{ijk})/\sqrt{-g}$ is the spin tensor.

Varying the Lagrangian with respect to the metric tensor g_{ik} gives the Einstein equations with the Ricci tensor. They can be put into a GR form with the **modified energy–momentum tensor**:

$$G^{ik} = \kappa T^{ik} + \kappa^2 \left(-s^{ij}{}_{[l} s^{kl}{}_{j]} - \frac{1}{2} s^{ijl} s^k{}_{jl} + \frac{1}{4} s^{jli} s_{jl}{}^k + \frac{1}{8} g^{ik} (-4s^l{}_{j[m} s^{jm}{}_{l]} + s^{jlm} s_{jlm}) \right). \quad 2$$

Spin fluid

Dirac spinors, representing **fermions**, couple to torsion through the covariant derivative in the Lagrangian and therefore are the **source of torsion**. At macroscopic scales, Dirac fields can be averaged and described as a spin fluid:

$$s_{ij}{}^k = s_{ij}u^k, \quad s_{ij}u^j = 0.$$

The terms in the effective energy–momentum tensor that are quadratic in the spin tensor do not vanish after averaging:

$$G^{ij} = \kappa \left(\epsilon - \frac{1}{4} \kappa s^2 \right) u^i u^j - \kappa \left(p - \frac{1}{4} \kappa s^2 \right) (g^{ij} - u^i u^j),$$

where

$$s^2 = \frac{1}{2} s_{ij} s^{ij} > 0 \propto n_f^2$$

is the averaged square of the spin density.

The Einstein–Cartan equations for a spin fluid are therefore equivalent to the GR Einstein equations for a perfect fluid with:

$$\tilde{\epsilon} = \epsilon - \alpha n_f^2, \quad \tilde{p} = p - \alpha n_f^2,$$

where ϵ and p are the thermodynamic energy density and pressure, n_f is the number density of fermions, and $\alpha = \kappa (\hbar c)^2 / 32$ with $\kappa = 8\pi G / c^4$.

Gravitational collapse of fluid sphere

A spherically symmetric gravitational field is given by the Tolman metric:

$$ds^2 = e^{\nu(\tau,R)} c^2 d\tau^2 - e^{\lambda(\tau,R)} dR^2 - e^{\mu(\tau,R)} (d\theta^2 + \sin^2\theta d\phi^2),$$

where ν , λ , and μ are functions of a time coordinate τ and a radial coordinate R . Coordinate transformations $\tau \rightarrow \tilde{\tau}(\tau)$ and $R \rightarrow \tilde{R}(R)$ do not change the form of the metric.

The components of the Einstein tensor corresponding to this metric that do not vanish identically are:

$$\begin{aligned} G_0^0 &= -e^{-\lambda} \left(\mu'' + \frac{3\mu'^2}{4} - \frac{\mu'\lambda'}{2} \right) + \frac{e^{-\nu}}{2} \left(\dot{\lambda}\dot{\mu} + \frac{\dot{\mu}^2}{2} \right) + e^{-\mu}, \\ G_1^1 &= -\frac{e^{-\lambda}}{2} \left(\frac{\mu'^2}{2} + \mu'\nu' \right) + e^{-\nu} \left(\ddot{\mu} - \frac{\dot{\mu}\dot{\nu}}{2} + \frac{3\dot{\mu}^2}{4} \right) + e^{-\mu}, \\ G_2^2 &= G_3^3 = -\frac{e^{-\nu}}{4} (\dot{\lambda}\dot{\nu} + \dot{\mu}\dot{\nu} - \dot{\lambda}\dot{\mu} - 2\ddot{\lambda} - \dot{\lambda}^2 - 2\ddot{\mu} - \dot{\mu}^2) \\ &\quad - \frac{e^{-\lambda}}{4} (2\nu'' + \nu'^2 + 2\mu'' + \mu'^2 - \mu'\lambda' - \nu'\lambda' + \mu'\nu'), \\ G_0^1 &= \frac{e^{-\lambda}}{2} (2\dot{\mu}' + \dot{\mu}\mu' - \dot{\lambda}\mu' - \dot{\mu}\nu'), \end{aligned}$$

where a dot denotes differentiation with respect to $c\tau$ and a prime denotes differentiation with respect to R .

Gravitational collapse of fluid sphere

In the comoving frame of reference, the spatial components of the four-velocity u^μ vanish. The nonzero components of the energy–momentum tensor for a spin fluid, $T_{\mu\nu} = (\tilde{\epsilon} + \tilde{p})u_\mu u_\nu - \tilde{p}g_{\mu\nu}$, are: $T_0^0 = \tilde{\epsilon}$, $T_1^1 = T_2^2 = T_3^3 = -\tilde{p}$. The Einstein field equations $G_\nu^\mu = \kappa T_\nu^\mu$ in this frame of reference are:

$$G_0^0 = \kappa\tilde{\epsilon}, \quad G_1^1 = G_2^2 = G_3^3 = -\kappa\tilde{p}, \quad G_0^1 = 0.$$

The covariant conservation of the energy–momentum tensor gives

$$\dot{\lambda} + 2\dot{\mu} = -\frac{2\dot{\tilde{\epsilon}}}{\tilde{\epsilon} + \tilde{p}}, \quad \nu' = -\frac{2\tilde{p}'}{\tilde{\epsilon} + \tilde{p}},$$

where the constants of integration depend on the allowed transformations $\tau \rightarrow \tilde{\tau}$ and $R \rightarrow \tilde{R}$.

If the pressure is homogeneous (no pressure gradients), then $\tilde{p}' = 0$, which gives $\nu' = 0$. Therefore, $\nu = \nu(\tau)$ and a transformation $\tau \rightarrow \tilde{\tau}$ can bring ν to zero and $g_{00} = e^\nu$ to 1. The system of coordinates becomes synchronous. Defining $r(\tau, R) = e^{\mu/2}$ turns the metric into

$$ds^2 = c^2 d\tau^2 - e^{\lambda(\tau, R)} dR^2 - r^2(\tau, R)(d\theta^2 + \sin^2\theta d\phi^2).$$

Every particle in a collapsing fluid sphere is represented by a value of R that ranges from 0 (at the center) to R_0 (at the surface).

Gravitational collapse of fluid sphere

The Einstein field equations reduce to

$$\begin{aligned}\kappa\tilde{\epsilon} &= -\frac{e^{-\lambda}}{r^2}(2rr'' + r'^2 - rr'\lambda') + \frac{1}{r^2}(r\dot{r}\dot{\lambda} + \dot{r}^2 + 1), \\ -\kappa\tilde{p} &= \frac{1}{r^2}(-e^{-\lambda}r'^2 + 2r\ddot{r} + \dot{r}^2 + 1), \\ -2\kappa\tilde{p} &= -\frac{e^{-\lambda}}{r}(2r'' - r'\lambda') + \frac{\dot{r}\dot{\lambda}}{r} + \ddot{\lambda} + \frac{1}{2}\dot{\lambda}^2 + \frac{2\ddot{r}}{r}, \\ 2\dot{r}' - \dot{\lambda}r' &= 0.\end{aligned}$$

Integrating the last equation gives

$$e^\lambda = \frac{r'^2}{1 + f(R)}, \quad (1)$$

where f is a function of R satisfying a condition $1 + f > 0$ (see Landau & Lifshitz, *The Classical Theory of Fields*). Substituting this relation into the second field equation gives $2r\ddot{r} + \dot{r}^2 - f = -\kappa\tilde{p}r^2$, which is integrated to

$$\dot{r}^2 = f(R) + \frac{F(R)}{r} - \frac{\kappa}{r} \int \tilde{p}r^2 dr,$$

where F is a positive function of R .

Gravitational collapse of fluid sphere

Substituting the last two equations into the first field equation gives a relation $\kappa(\tilde{\epsilon} + \tilde{p}) = F'(R)/(r^2 r')$, leading to

$$\dot{r}^2 = f(R) + \frac{\kappa}{r} \int_0^R \tilde{\epsilon} r^2 r' dR. \quad (2)$$

If the mass of the sphere is M , then the Schwarzschild radius $r_g = 2GM/c^2$ of the black hole that forms from the sphere is equal to

$$r_g = \kappa \int_0^{R_0} \tilde{\epsilon} r^2 r' dR.$$

These two equations give $\dot{r}^2(\tau, R_0) = f(R_0) + r_g/r(\tau, R_0)$. If $r_0 = r(0, R_0)$ is the initial radius of the sphere and the sphere is initially at rest, then $\dot{r}(0, R_0) = 0$. Consequently, the value of R_0 is determined by

$$f(R_0) = -\frac{r_g}{r_0}.$$

Substituting $r = e^{\mu/2}$ and (1) into the first conservation law gives the first law of thermodynamics for the effective energy density and pressure:

$$\frac{d}{d\tau}(\tilde{\epsilon} r^2 r') + \tilde{p} \frac{d}{d\tau}(r^2 r') = 0. \quad (3) \quad 7$$

Collapse of spin fluid sphere

If we assume that the spin fluid is composed of an ultrarelativistic matter in kinetic equilibrium, then $\epsilon = h_\star T^4$, $p = \epsilon/3$, and $n_f = h_{nf} T^3$, where T is the temperature of the fluid, $h_\star = (\pi^2/30)(g_b + (7/8)g_f)k^4/(\hbar c)^3$, and $h_{nf} = (\zeta(3)/\pi^2)(3/4)g_f k^3/(\hbar c)^3$. For standard-model particles, $g_b = 29$ and $g_f = 90$. The effective energy density and pressure are thus:

$$\tilde{\epsilon} = h_\star T^4 - \alpha h_{nf}^2 T^6, \quad \tilde{p} = \frac{1}{3} h_\star T^4 - \alpha h_{nf}^2 T^6.$$

Since the pressure has no gradient, the temperature only depends on τ , and so does the energy density. This scenario describes a homogeneous sphere. The first law of thermodynamics (3) gives

$$r^2 r' T^3 = g(R), \tag{4}$$

where g is a function of R . Putting this relation into (2) gives

$$\dot{r}^2 = f(R) + \frac{\kappa}{r} (h_\star T^4 - \alpha h_{nf}^2 T^6) \int_0^R r^2 r' dR. \tag{5}$$

Equations (4) and (5) give the function $r(\tau, R)$, which with (1) gives $\lambda(\tau, R)$. The integration of (5) also contains the initial value $\tau_0(R)$. The metric therefore depends on three arbitrary functions: $f(R)$, $g(R)$, and $\tau_0(R)$.

Collapse of spin fluid sphere

We seek a solution of (4) and (5) as

$$f(R) = -\sin^2 R, \quad r(\tau, R) = a(\tau) \sin R, \quad (6)$$

where $a(\tau)$ is a nonnegative function of τ . For this choice of functions, (4) gives $a^3 T^3 \sin^2 R \cos R = g(R)$, in which separation of the variables τ and R leads to

$$g(R) = \text{const} \cdot \sin^2 R \cos R, \quad a^3 T^3 = \text{const}.$$

Consequently,

$$aT = a_0 T_0, \quad \frac{\dot{T}}{T} + \frac{H}{c} = 0, \quad (7)$$

where $a_0 = a(\tau = 0)$ and $T_0 = T(\tau = 0)$ are the values at the initial time, and $H = c\dot{a}/a$. Substituting (6) into (5) gives

$$\dot{a}^2 + 1 = \frac{\kappa}{3} (h_* T^4 - \alpha h_{nf}^2 T^6) a^2, \quad (8)$$

which has a form of the Friedmann equation for the scale factor a as a function of the cosmic time τ in a closed, homogeneous universe. The quantity H is the Hubble parameter of this universe. Using (7) in (8) yields

$$\dot{a}^2 = -1 + \frac{\kappa}{3} \left(\frac{h_* T_0^4 a_0^4}{a^2} - \frac{\alpha h_{nf}^2 T_0^6 a_0^6}{a^4} \right). \quad (9)$$

Collapse of spin fluid sphere

The relations (6) determine the constants:

$$\sin R_0 = \left(\frac{r_g}{r_0}\right)^{1/2}, \quad a(0) = \left(\frac{r_0^3}{r_g}\right)^{1/2}.$$

Substituting the initial values $a(0)$ and $\dot{a}(0) = 0$ into (8), in which the second term on the right-hand side is negligible, gives $Mc^2 = (4\pi/3)r_0^3 h_* T_0^4$. This relation indicates the equivalence of mass and energy of a fluid sphere with radius r_0 and determines T_0 . An event horizon for the entire sphere forms when $r(\tau, R_0) = r_g$, which is equivalent to $a = (r_g r_0)^{1/2}$. Equation (9) has two turning points, $\dot{a} = 0$, if

$$\frac{r_0^3}{r_g} > \frac{3\pi G \hbar^4 h_{nf}^4}{8h_*^3} \sim l_{\text{Planck}}^2,$$

which is satisfied for astrophysical systems that form black holes.

Substituting (6) into (1) gives $e^{\lambda(\tau, R)} = a^2$. Consequently, the square of an infinitesimal interval in the interior of a collapsing spin fluid is given by

$$ds^2 = c^2 d\tau^2 - a^2(\tau) dR^2 - a^2(\tau) \sin^2 R (d\theta^2 + \sin^2 \theta d\phi^2).$$

This metric has a form of the closed Friedmann–Lemaître–Robertson–Walker metric and describes a part of a **closed universe** with $0 \leq R \leq R_0$ (like dust).

Nonsingular bounce

Equation (9) can be solved analytically in terms of an elliptic integral of the second kind, giving the function $a(\tau)$ and then $r(\tau, R) = a(\tau) \sin R$:

$$\dot{a}^2 = -1 + \frac{\kappa}{3} \left(\frac{h_* T_0^4 a_0^4}{a^2} - \frac{\alpha h_{nf}^2 T_0^6 a_0^6}{a^4} \right). \quad (9)$$

The **value of a never reaches zero** because as a decreases, the right-hand side of (9) becomes negative, contradicting the left-hand side. The change of the sign occurs when $a < (r_g r_0)^{1/2}$, that is, after the event horizon forms. Consequently, all particles with $R > 0$ fall within the event horizon but never reach $r = 0$ (the only particle at the center is the particle that is initially at the center, with $R = 0$). **A singularity is replaced with a nonsingular bounce.** Nonzero values of a give finite values of T and thus finite values of ϵ , p , and n_f .

After the bounce, the matter expands on the other side of the event horizon as a **new universe**. This universe has a closed geometry (constant positive curvature). The quantity $a(\tau)$ is the scale factor. The universe is oscillatory: the value of a oscillates between the two turning points. The value of R_0 does not change. A turning point at which $\ddot{a} > 0$ is a bounce, and a turning point at which $\ddot{a} < 0$ is a crunch. This universe has an infinite number of identical cycles.

N. Popławski, *Astrophys. J.* 832, 96 (2016).

G. Unger and N. Popławski, *Astrophys. J.* 870, 78 (2019).

Nonsingular bounce

The Raychaudhuri equation for a congruence of geodesics without four-acceleration and rotation is $d\theta/ds = -\theta^2/3 - 2\sigma^2 - R_{\mu\nu}u^\mu u^\nu$, where θ is the expansion scalar, σ^2 is the shear scalar, and $R_{\mu\nu}$ is the Ricci tensor. For a spin fluid, the last term in this equation is equal to $-\kappa(\tilde{\epsilon} + 3\tilde{p})/2$. Consequently, the necessary and sufficient condition for avoiding a singularity in a black hole is $-\kappa(\tilde{\epsilon} + 3\tilde{p})/2 > 2\sigma^2$. For a relativistic spin fluid, $p = \epsilon/3$, this condition is equivalent to

$$2\kappa\alpha n_f^2 > 2\sigma^2 + \kappa\epsilon. \quad (10)$$

Without torsion, the left-hand side of (10) would be absent and this inequality could not be satisfied, resulting in a singularity. **Torsion may provide a necessary condition for preventing a singularity.** In the absence of shear, this condition would be also sufficient.

(Hehl, Trautman, Kopczyński, Tafel, Kuchowicz)

The presence of shear opposes the effects of torsion. The shear scalar σ^2 grows with decreasing a like $\sim a^{-6}$, which is the same power law as that for n_f^2 . Therefore, if the initial shear term dominates over the initial torsion term in (10), then it will dominate at later times during contraction and a singularity will form. To avoid a singularity if the shear is present, n_f^2 must grow faster than $\sim a^{-6}$. Consequently, **fermions must be produced** in a black hole during contraction.

Nonsingular bounce

The production rate of particles in a contracting or expanding universe can be phenomenologically given by

$$\frac{1}{c\sqrt{-g}} \frac{d(\sqrt{-g}n_f)}{dt} = \frac{\beta H^4}{c^4}, \quad (11)$$

where $g = -a^6 \sin^4 R \sin^2 \theta$ is the determinant of the metric tensor and β is a nondimensional production rate. With particle production, the second equation in (7) turns into

$$\frac{\dot{T}}{T} = \frac{H}{c} \left(\frac{\beta H^3}{3c^3 h_{nf} T^3} - 1 \right). \quad (12)$$

Particle production changes the power law $n_f(a)$:

$$n_f \sim a^{-(3+\delta)},$$

where δ varies with τ . Putting this relation into (11) gives

$$\delta \sim -a^\delta \dot{a}^3.$$

During contraction, $\dot{a} < 0$ and thus $\delta > 0$. The term $n_f^2 \sim a^{-6-2\delta}$ grows faster than $\sigma^2 \sim a^{-6}$ and a singularity is avoided. **Particle production and torsion together reverse the effects of shear, generating a bounce.**

The universe in a black hole

The dynamics of the nonsingular, relativistic universe in a black hole is described by equations (8) and (12):

$$\dot{a}^2 + 1 = \frac{\kappa}{3}(h_{\star}T^4 - \alpha h_{nf}^2 T^6)a^2, \quad \frac{\dot{T}}{T} = \frac{H}{c} \left(\frac{\beta H^3}{3c^3 h_{nf} T^3} - 1 \right),$$

where $H = c\dot{a}/a$. These equations, with the initial conditions $a(0) = (r_0^3/r_g)^{1/2}$ and $\dot{a}(0) = 0$, give the functions $a(\tau)$ and $T(\tau)$.

The shear would enter the right-hand side of the first equation as an additional positive term that is proportional to a^{-4} . When the universe becomes nonrelativistic, the term $h_{\star}T^4$ changes into a positive term proportional to a^{-1} . The cosmological constant enters as a positive term proportional to a^2 .

Particle production increases the maximum size of the scale factor that is reached at a crunch. Consequently, a new cycle is larger and lasts longer than the previous cycle. R_0 is given by $\sin^3 R_0 = r_g/a(0)$, where $a(0)$ is the maximum scale factor in the first cycle. Since the maximum scale factor in the next cycle is larger, the value of $\sin R_0$ decreases. As cycles proceed, R_0 approaches π (the value for a completely closed universe).

A parent black hole creating a new, baby universe becomes an **Einstein–Rosen bridge** (unidirectional wormhole) to that universe.

Closed Universe

If the Universe is closed, it is analogous to the 2-dimensional surface of a 3-dimensional sphere. The Universe would be mathematically the 3-dimensional hypersurface of a 4-dimensional hypersphere.

The 3-dimensional space in which the balloon expands is not analogous to any higher dimensional space. Points off the surface of the balloon are not in the Universe in this analogy.

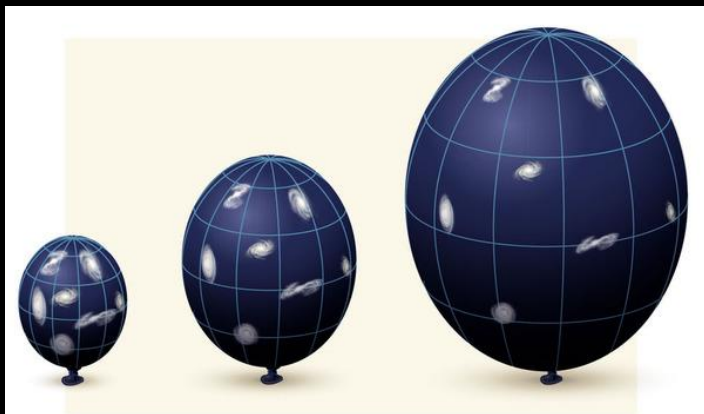


Image credit: One-Minute Astronomer

The balloon radius = **scale factor** a .

The Universe expands (Hubble law).

The Universe may be finite (closed) or infinite (flat or open).

Inflation

During expansion ($H > 0$), if β is too big, then the right-hand side could become positive:

$$\frac{\dot{T}}{T} = \frac{H}{c} \left(\frac{\beta H^3}{3c^3 h_{nf} T^3} - 1 \right).$$

In this case, the temperature would grow with increasing a , which would lead to eternal inflation. Consequently, there is an upper limit to the production rate: the maximum of the function $(\beta H^3)/(3c^3 h_{nf} T^3)$ must be lesser than 1.

If $(\beta H^3)/(3c^3 h_{nf} T^3)$ increases after a bounce to a value that is slightly lesser than 1, then T would become approximately constant. Accordingly, H would be also nearly constant and the scale factor a would grow exponentially, generating inflation. Since the energy density would be also nearly constant, the universe would produce enormous amounts of matter and entropy. Such an expansion would last until the right-hand side of drops below 1. Consequently, inflation would last a finite period of time. After this period, the effects of torsion weaken and the universe smoothly enters the radiation-dominated expansion, followed by the matter-dominated expansion.

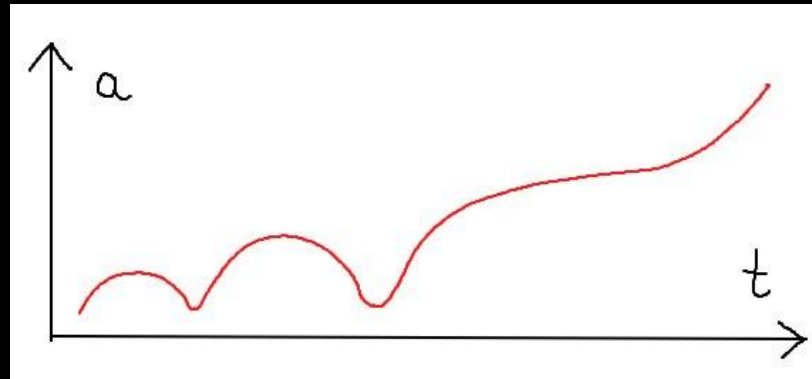
Torsion and particle production can generate **finite inflation** without scalar fields and reheating.

N. J. Popławski, Phys. Lett. B 694, 181 (2010).

N. Popławski, Astrophys. J. 832, 96 (2016).

Dark energy ceasing oscillations

If quantum effects in the gravitational field near a bounce do not produce enough matter, then the closed Universe reaches the maximum size and then contracts to another bounce, beginning the new cycle. Because of matter production, a new cycle reaches larger size and last longer than the previous cycle.



When the Universe reaches a size at which the cosmological constant is dominating, then it avoids another contraction and starts expanding to infinity.

Every black hole creates a new universe? Our Universe originated in a black hole?

Every black hole may create a new, closed, baby universe (Novikov, Pathria, Hawking, Smolin, NP).

Accordingly, our Universe may be closed and may have born in the interior of a black hole existing in a parent universe.

NP, PLB 694, 181 (2010)

This hypothesis could solve the black hole information paradox: the information goes through the Einstein-Rosen bridge to the baby universe on the other side of the black hole's event horizon.

The motion through an event horizon is unidirectional: it defines the past and future. Time asymmetry at the event horizon may induce time asymmetry everywhere in the baby universe and explain why time flows in one direction.

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Summary - our Universe in a black hole

- The conservation law for total angular momentum in curved spacetime, consistent with Dirac equation, requires torsion. The simplest theory with torsion, Einstein-Cartan gravity, has the same Lagrangian as GR, but the affine connection contains the torsion tensor, generated by spin.
- Gravitational collapse of a sphere of a spin fluid creates an event horizon. The matter within the horizon collapses to extremely high densities, at which torsion acts like gravitational repulsion.
- Classically, without shear, torsion prevents a singularity and replaces it with a nonsingular bounce. With shear, torsion prevents a singularity if the number of fermions increases during contraction via quantum particle production.
- Particle production during expansion produces enormous amounts of matter and can generate a finite period of inflation. The resulting closed universe on the other side of the event horizon may have several bounces. Such a universe is oscillatory, with each cycle larger in size than the previous cycle, until it reaches the cosmological size and expands indefinitely.