

Torsional regularization and finite bare charge

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Problems of general relativity

General relativity describes gravity as curvature of spacetime.

- Singularities: points with infinite density of matter.
- Incompatible with quantum mechanics. We need quantum gravity. It may resolve the singularity problem.
- Field equations contain the conservation of orbital angular momentum, contradicting Dirac equation which gives the conservation of total angular momentum (orbital + spin) and allows spin-orbit exchange in QM.

Simplest extension of GR to include QM spin:

Einstein-Cartan theory. It also eliminates the singularity problem.

Einstein-Cartan-Sciama-Kibble gravity

- Spacetime has curvature and **torsion**.

$$S^k{}_{ij} = \Gamma^k{}_{[ij]}$$

- Lagrangian density is proportional to curvature scalar (as in GR).
- Cartan equations:

Torsion is proportional to **spin** density of fermions. ECSK differs significantly from GR at densities $> 10^{45}$ kg/m³; passes all tests.

$$S_{jik} - S_i g_{jk} + S_k g_{ji} = -\frac{1}{2} \kappa S_{ikj}$$

arXiv.org > gr-qc > arXiv:0911.0334

- Einstein equations:

Curvature is proportional to **energy and momentum** density.

$$G^{ik} = \kappa T^{ik} + \frac{1}{2} \kappa^2 \left(s^{ij}{}_j s^{kl}{}_l - s^{ij}{}_l s^{kl}{}_j - s^{ijl} s^k{}_{jl} + \frac{1}{2} s^{jli} s_{jl}{}^k + \frac{1}{4} g^{ik} (2s_j{}^l{}_m s^{jm}{}_l - 2s_j{}^l{}_l s^{jm}{}_m + s^{jlm} s_{jlm}) \right)$$

Universe with spin fluid

Dirac particles can be averaged macroscopically as a spin fluid.

Einstein-Cartan equations for a homogeneous and isotropic Universe become Friedmann equations for the scale factor a .

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa\left(\epsilon - \frac{1}{4}\kappa s^2\right)a^2$$
$$\frac{\dot{a}^2 + 2a\ddot{a}}{c^2} + 1 = -\kappa\left(p - \frac{1}{4}\kappa s^2\right)a^2$$

$$s^2 = \frac{1}{8}(\hbar cn)^2$$

Spin and torsion modify the energy density and pressure with a **negative** term proportional to the square of the fermion number density n , which acts like **repulsive gravity** and prevents singularities. The Big Bang is replaced by a non-singular Big Bounce.

Problems of quantum field theory

- Ultraviolet divergence: Feynman diagrams involve divergent integrals in the four-momentum space arising from high-energy contributions.
- This unphysical result requires regularization: a mathematical method of turning singular quantities into finite quantities. Most common: adding fictitious particles, changing dimensions.
- Renormalization: the original (bare) values of mass and charge absorb divergent terms, giving the measured (dressed) values.
- Dirac was critical about renormalization and expected a realistic regularization based on the principles of physics.
- Solution: **torsional regularization**, renormalization is finite.

Torsion and noncommutativity of momentum

- Consider two infinitesimal four-vectors dx and dx' .
- In the presence of torsion the parallel transport of dx along dx' and the parallel transport of dx' along dx do not form a closed parallelogram:

$$\delta dx^i = -\Gamma_{jk}^i dx^j dx'^k$$

$$\delta dx'^i = -\Gamma_{jk}^i dx^j dx'^k$$

$$\delta dx'^i - \delta dx^i = -S^i_{jk} dx^j dx'^k$$

- Since the momentum is a generator of translation, described by the parallel transport, its operator in quantum mechanics is given by the covariant derivative:

$$p_k = i\hbar \nabla_k$$

- In the presence of torsion, translations do not commute and therefore the four-momentum components do not commute:

$$[p_i, p_j] = 2i\hbar S^k_{ij} p_k$$

Integration in momentum space becomes summation over momentum eigenstates

- The classical and quantum partition functions in statistical physics:

$$\int dq \int dp f(H(q,p)) \leftrightarrow 2\pi \sum_{\text{eigenstates}} f(E) |[q,p]|$$

- One can choose locally a frame of reference in which only the space momentum components do not commute:

$$[p_x, p_y] = iQp_z, \quad [p_y, p_z] = iQp_x, \quad [p_z, p_x] = iQp_y$$

$$Q = -2\hbar A^0$$

$$A^i = \frac{1}{6} \epsilon^{ijkl} S_{jkl}$$

- Einstein-Cartan gravity gives: $Q = Up^3$ (U is const $\sim M_{pl}^{-2}$)

- We obtain a relation analogous to the angular momentum:

$$[n_x, n_y] = in_z, \quad [n_y, n_z] = in_x, \quad [n_z, n_x] = in_y$$

$$\mathbf{n} = \frac{\mathbf{p}}{Q}$$

Integration in momentum space becomes summation over momentum eigenstates

- We propose that the integration in n -space satisfying

$$[n_x, n_y] = in_z, \quad [n_y, n_z] = in_x, \quad [n_z, n_x] = in_y$$

- Is replaced with the summation:

$$\int dn_x \int dn_y \int dn_z f(\mathbf{n}^2) \rightarrow 4\pi \sum_{\text{eigenstates}} f(\mathbf{n}^2) |n_z|$$

$$\rightarrow 4\pi \sum_{l=1}^{\infty} \sum_{m=-l}^l f(\mathbf{n}^2) |m| = 4\pi \sum_{l=1}^{\infty} f(\mathbf{n}^2) l(l+1)$$

- If the integral is finite, the corresponding sum is almost equal.
- **Torsional regularization – NP, arXiv:1712.09997**

Integration in momentum space becomes summation over momentum eigenstates

- Apply TR to a logarithmically divergent integral:

$$\int \frac{d^4 p}{(p^2 + \mu^2)^2} = \int \frac{dp_0 d\mathbf{p}}{(p^2 + \mu^2)^2} = \int \frac{dp_0 J d\mathbf{n}}{(p^2 + \mu^2)^2} \rightarrow 4\pi \int_{-\infty}^{\infty} dp_0 \sum_{l=1}^{\infty} \frac{J}{(p^2 + \mu^2)^2} l(l+1)$$

$$J = \partial(p_x, p_y, p_z) / \partial(n_x, n_y, n_z)$$

$$p^2 = p_0^2 + U^2 n^2 p^6$$

$$\frac{\partial p}{\partial n_x} = \frac{U^2 p^5 n_x}{1 - 3U^2 n^2 p^4}$$

$$\frac{\partial p_x}{\partial n_x} = \frac{\partial(Q n_x)}{\partial n_x} = Q + 3U n_x p^2 \frac{\partial p}{\partial n_x}$$

$$\frac{\partial p_x}{\partial n_y} = \frac{\partial(Q n_x)}{\partial n_y} = 3U n_x p^2 \frac{\partial p}{\partial n_y}$$

$$dp_0/dp = (1 - 3U^2 n^2 p^4) / (1 - U^2 n^2 p^4)^{1/2}$$

$$n = \sqrt{l(l+1)}$$

$$J = \det \begin{pmatrix} \partial p_x / \partial n_x & \partial p_x / \partial n_y & \partial p_x / \partial n_z \\ \partial p_y / \partial n_x & \partial p_y / \partial n_y & \partial p_y / \partial n_z \\ \partial p_z / \partial n_x & \partial p_z / \partial n_y & \partial p_z / \partial n_z \end{pmatrix} = \frac{Q^3}{1 - 3U^2 n^2 p^4}$$

Torsion eliminates ultraviolet divergence

$$\begin{aligned}
 & 4\pi \int_{-\infty}^{\infty} dp_0 \sum_{l=1}^{\infty} \frac{Q^3 n^2}{(1 - 3U^2 n^2 p^4)(p^2 + \mu^2)^2} = 4\pi \int dp \frac{dp_0}{dp} \sum_{l=1}^{\infty} \frac{Q^3 n^2}{(1 - 3U^2 n^2 p^4)(p^2 + \mu^2)^2} \\
 & = 4\pi \int_{-1/\sqrt{Un}}^{1/\sqrt{Un}} dp \sum_{l=1}^{\infty} \frac{Q^3 n^2}{(1 - U^2 n^2 p^4)^{1/2} (p^2 + \mu^2)^2} = 8\pi \int_0^{1/\sqrt{Un}} dp \sum_{l=1}^{\infty} \frac{U^3 p^9 n^2}{(1 - U^2 n^2 p^4)^{1/2} (p^2 + \mu^2)^2} \\
 & = 8\pi \int_0^1 d\xi \sum_{l=1}^{\infty} \frac{U^3 \xi^9 n^2 (Un)^{-5}}{(1 - \xi^4)^{1/2} [\xi^2 / (Un) + \mu^2]^2} = 8\pi \int_0^1 d\xi \sum_{l=1}^{\infty} \frac{\xi^9 n^{-1}}{(1 - \xi^4)^{1/2} [\xi^2 + U\mu^2 n]^2} \\
 & = 4\pi \int_0^1 d\zeta \sum_{l=1}^{\infty} \frac{\zeta^4 n^{-1}}{(1 - \zeta^2)^{1/2} [\zeta + U\mu^2 n]^2} = 4\pi \sum_{l=1}^{\infty} \int_0^{\pi/2} d\phi \frac{\sin^4 \phi n^{-1}}{[\sin \phi + U\mu^2 n]^2} \\
 & = 4\pi \sum_{l=1}^{\infty} \int_0^{\pi/2} d\phi \frac{\sin^4 \phi [l(l+1)]^{-1/2}}{[\sin \phi + U\mu^2 \sqrt{l(l+1)}]^2}, \quad Unp^2 = \xi^2 = \zeta = \sin \phi
 \end{aligned}$$

The logarithmically divergent integral is replaced with a sum that converges as l^{-3} .

Torsion eliminates ultraviolet divergence

This procedure can be generalized to tensor integrals:

$$\begin{aligned}
 \int \frac{d^4 p}{(p^2 + \mu^2)^s} &\rightarrow 8\pi \int_0^{1/\sqrt{Un}} dp \sum_{l=1}^{\infty} \frac{U^3 p^9 n^2}{(1 - U^2 n^2 p^4)^{1/2} (p^2 + \mu^2)^s} \\
 &= 8\pi \int_0^1 d\xi \sum_{l=1}^{\infty} \frac{U^3 \xi^9 n^2 (Un)^{-5}}{(1 - \xi^4)^{1/2} [\xi^2/(Un) + \mu^2]^s} = 8\pi \int_0^1 d\xi \sum_{l=1}^{\infty} \frac{U^{s-2} \xi^9 n^{s-3}}{(1 - \xi^4)^{1/2} [\xi^2 + U\mu^2 n]^s} \\
 &= 4\pi \int_0^1 d\zeta \sum_{l=1}^{\infty} \frac{U^{s-2} \zeta^4 n^{s-3}}{(1 - \zeta^2)^{1/2} [\zeta + U\mu^2 n]^s} = 4\pi U^{s-2} \sum_{l=1}^{\infty} \int_0^{\pi/2} d\phi \frac{\sin^4 \phi n^{s-3}}{[\sin \phi + U\mu^2 n]^s} \\
 &= 4\pi U^{s-2} \sum_{l=1}^{\infty} \int_0^{\pi/2} d\phi \frac{\sin^4 \phi [l(l+1)]^{(s-3)/2}}{[\sin \phi + U\mu^2 \sqrt{l(l+1)}]^s}.
 \end{aligned}$$

$$\int d^4 p \frac{\partial}{\partial p_\nu} \left(\frac{p^\mu}{(p^2 + \Delta)^s} \right) = \int d^4 p \frac{\delta^{\mu\nu}}{(p^2 + \Delta)^s} - 2s \int d^4 p \frac{p^\mu p^\nu}{(p^2 + \Delta)^{s+1}}$$

$$\int d^4 p \frac{p^\mu p^\nu}{(p^2 + \Delta)^s} = \frac{\delta^{\mu\nu}}{2(s-1)} \int d^4 p \frac{1}{(p^2 + \Delta)^{s-1}}$$

Vacuum polarization

The vacuum polarization tensor is gauge invariant:

$$\begin{aligned}\Pi_{\text{bubble}}^{\mu\nu}(q) &= -\frac{\alpha_0}{\pi^3} \int d^4 p_E \int_0^1 dx \frac{-2p_E^\mu p_E^\nu + p_E^2 \delta^{\mu\nu} + \Delta \delta^{\mu\nu} + 2(q^2 g^{\mu\nu} - q^\mu q^\nu)x(1-x)}{(p_E^2 + \Delta)^2} \\ &= -\frac{2\alpha_0}{\pi^3} \int d^4 p_E \int_0^1 dx \frac{x(1-x)}{(p_E^2 + \Delta)^2} (q^2 g^{\mu\nu} - q^\mu q^\nu) = \Pi(q^2) q^2 \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right),\end{aligned}$$

$$\Pi(q^2) = -\frac{2\alpha_0}{\pi^3} \int d^4 p_E \int_0^1 dx \frac{x(1-x)}{(p_E^2 + \Delta)^2}$$

$$\Delta = m^2 - q^2 x(1-x)$$

$$\begin{aligned}\Pi(q^2) &\rightarrow -\frac{8\alpha_0}{\pi^2} \sum_{l=1}^{\infty} \int_0^1 dx \int_0^{\pi/2} d\phi \frac{\sin^4 \phi n^{-1} x(1-x)}{[\sin \phi + U \Delta n]^2} \\ &= -\frac{8\alpha_0}{\pi^2} \sum_{l=1}^{\infty} \int_0^1 dx \int_0^{\pi/2} d\phi \frac{\sin^4 \phi [l(l+1)]^{-1/2} x(1-x)}{[\sin \phi + U \Delta \sqrt{l(l+1)}]^2}\end{aligned}$$

The sum-integral
in Π is finite.

Torsion makes bare charge finite

Renormalization of the electric charge:

$$\alpha = \frac{\alpha_0}{1 - \Pi(0)}$$

$$\alpha_{\text{run}} = \frac{\alpha_0}{1 - \Pi(q^2)}$$

Gives the bare electric charge of an electron:

$$e_0 = \frac{e}{(1 + \Pi_{\text{R}}(0))^{1/2}} = e \left[1 - \frac{8\alpha}{\pi^2} \sum_{l=1}^{\infty} \int_0^1 dx \int_0^{\pi/2} d\phi \frac{\sin^4 \phi n^{-1} x(1-x)}{[\sin \phi + Um^2 n]^2} \right]^{-1/2}$$

Including all charged fermions in Π gives the bare charge **-1.22 e**.
The running coupling constant is finite.

Accordingly, the bare fine structure constant is about 1/92.1.

NP, arXiv:1712.09997

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Summary

- The conservation law for total angular momentum (orbital + spin) in curved spacetime, consistent with Dirac equation, requires torsion.
- In the presence of torsion, the four-momentum operator components do not commute. The integration in the momentum space must be replaced with the summation over the momentum eigenvalues.
- The separation between the momentum eigenvalues increases with the magnitude of the momentum as a result of the Einstein-Cartan gravity. Consequently, ultraviolet divergent integrals turn into convergent sums.
- Torsion naturally regularizes ultraviolet divergence in QED. Renormalization in QED is finite, leading to a finite bare charge of an electron: $-1.22 e$.
- Future work: research how torsion affects the electroweak and strong interactions.