Universe in a Black Hole with Spin and Torsion

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Universo en un Agujero Negro con Espín y Torsión

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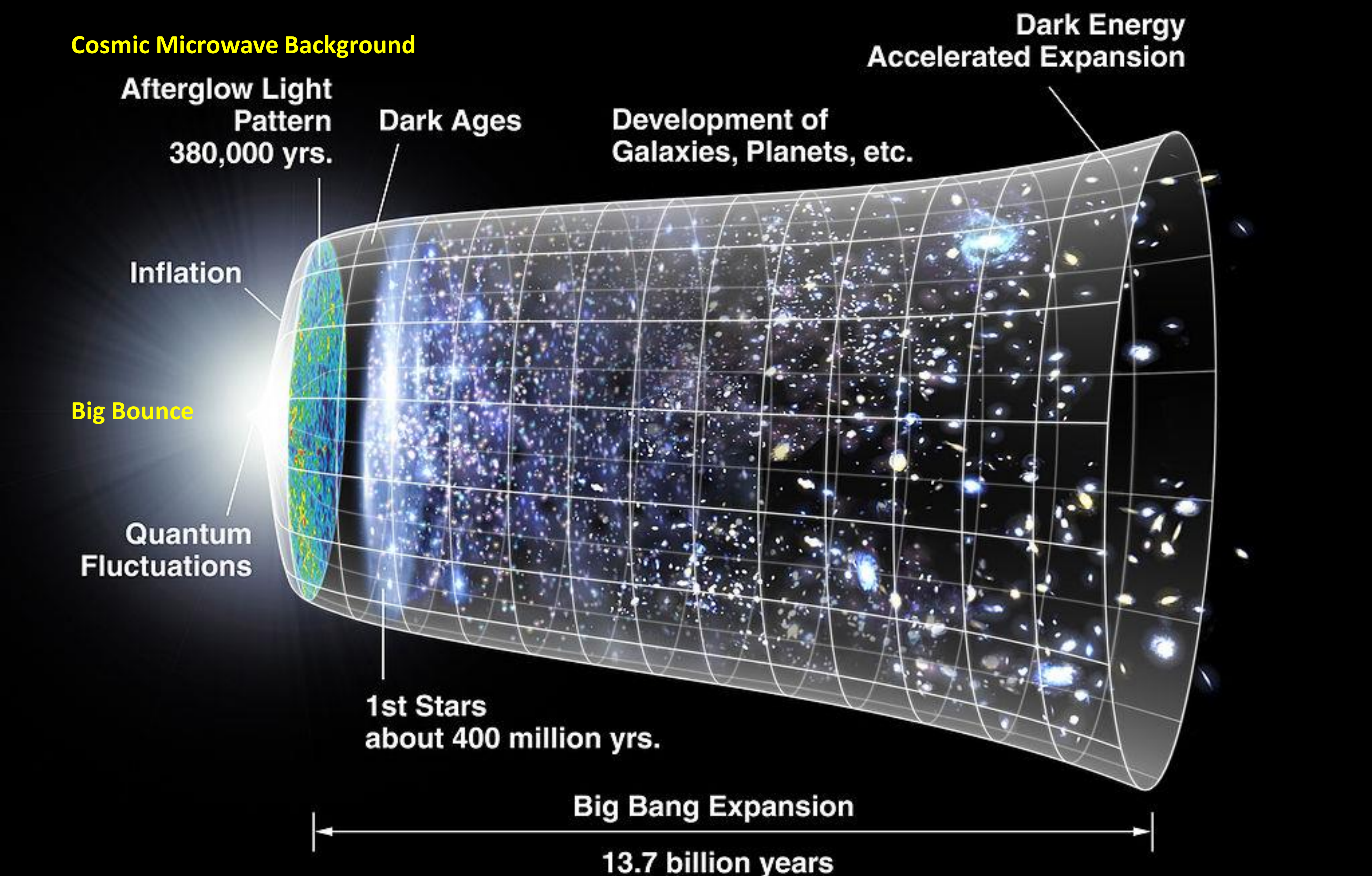
Abstract

We consider gravitational collapse of a spherically symmetric sphere of a fluid with spin and torsion into a black hole. We use the Tolman metric and the Einstein-Cartan field equations with a relativistic spin fluid as a source.

We show that gravitational repulsion of torsion prevents a singularity and replaces it with a nonsingular bounce. Quantum particle production during contraction strengthens torsion against opposing shear. Particle production during expansion can produce enormous amounts of matter and generate a finite period of inflation.

The resulting closed universe on the other side of the event horizon may have several bounces. Such a universe is oscillatory, with each cycle larger in size then the previous cycle, until it reaches the cosmological size and expands indefinitely. Our universe might have therefore originated from a black hole existing in another universe.



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How do we know the Big Bang happened?

- We can see the Universe expanding: galaxies look redder as they speed away (just as sirens sound lower pitched).

Hubble's law: speed of receding galaxies is proportional to their distances.

- Big Bang Nucleosynthesis (Gamow).

If the Universe is closed, the 2-dimensional surface of the balloon is an analog to our 3-dimensional space.

The 3-dimensional space in which the balloon expands is not analogous to any higher dimensional space. Points off the surface of the balloon are not in the Universe in this simple analogy.

The balloon radius = the scale factor *a*.

The Universe may be finite (closed) or infinite (flat or open).

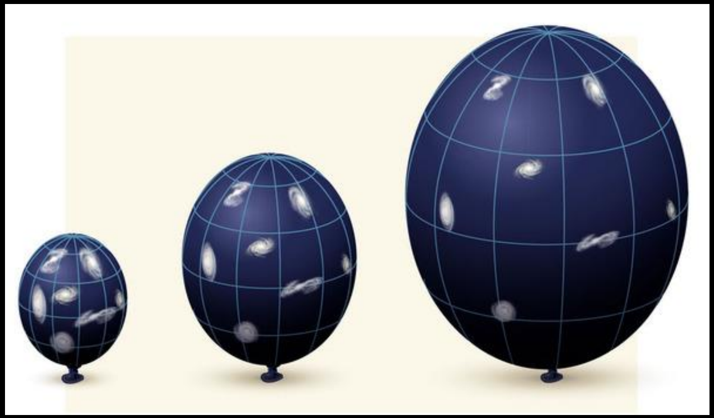


Image credit: One-Minute Astronomer

How do we know the Big Bang happened?

- We can observe the Cosmic Microwave Background. This electromagnetic radiation is a remnant from an early stage of the Universe. When protons and electrons combined (recombination) to form neutral hydrogen atoms that could not absorb photons, photons decoupled and the Universe became transparent . Currently, the CMB comes to us from all directions in the sky and has a temperature 2.725 K.

- The temperature anisotropy, about 2×10^{-5}, provides information about the dynamics of the early Universe.

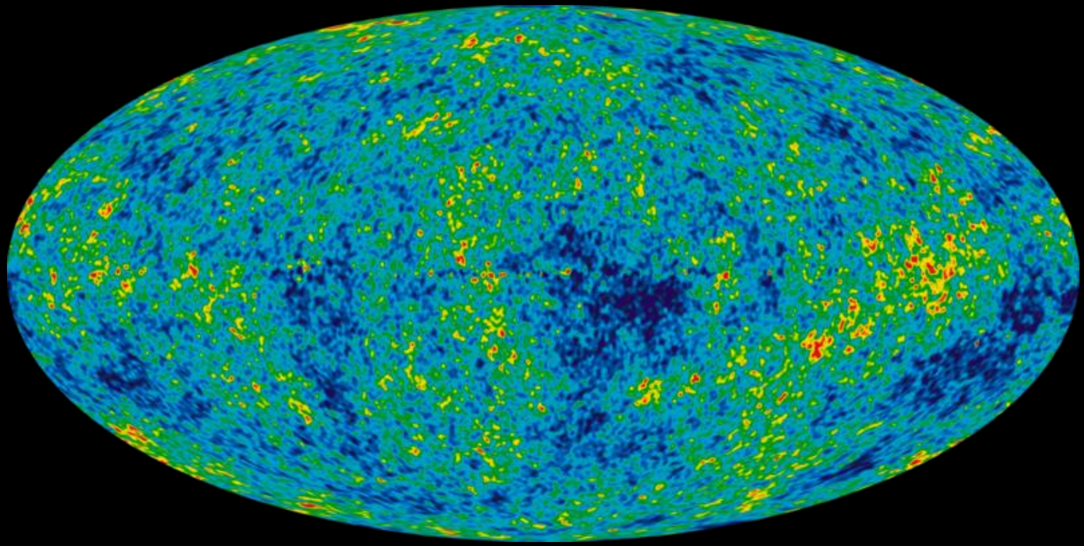


Image credit: Wikipedia

Problems of General Relativity

Cosmology is based on General Relativity, which describes gravity as curvature of spacetime.

- Singularities: points with infinite density of matter.
- Incompatible with quantum mechanics. We need quantum gravity. It may resolve the singularity problem.
- Field equations contain the conservation of orbital angular momentum, contradicting Dirac equation which gives the conservation of total angular momentum (orbital + spin) and allows spin-orbit exchange in QM.

Simplest extension of GR to include QM spin:
Einstein-Cartan theory. It also resolves the singularity problem.

Problems of cosmology

- Big-Bang singularity.
- What caused the Big Bang? What existed before?
- Horizon problem: different regions of the Universe have not contacted each other because of large distances between them, but they have the same physical properties (the Universe is homogeneous and isotropic). The temperature of the CMB is almost isotropic.
- Flatness problem: the Universe is nearly flat. The initial conditions of the Universe must have been fine-tuned.
- What was the initial condition for structure formation?

Problems of cosmology

- Inflation (Guth, Linde): exponential expansion of the early Universe) explains why the Universe appears flat, homogeneous, and isotropic.

- What caused inflation? Hypothetical scalar field: inflaton.

- Quantum fluctuations of inflaton are stretched to macroscopic scales and freeze in upon leaving the horizon. At the later stages of radiation- and matter-domination, they re-enter the horizon and set the initial condition for structure formation. Inflation predicts the observed spectrum of CMB anisotropies.

- How did inflation end? The large inflaton potential energy decays into Standard Model particles. Poorly understood. Eternal inflation may occur.

Einstein-Cartan-Sciama-Kibble gravity

- Einstein-Cartan theory replaces the Big Bang by a nonsingular **Big Bounce**. The dynamics after the bounce explains the flatness and horizon problems.

- Spacetime is equipped with torsion.
  Curvature – "bending" of spacetime
  Torsion – "twisting" of spacetime

- Bending a thin rod is more apparent than twisting. Effects of torsion are important only in extreme situations (in black holes and in the very early Universe).

NP, PLB 694, 181 (2010)

Einstein-Cartan-Sciama-Kibble gravity

- Torsion tensor is a field in addition to the metric. $S^{k}{}_{ij} = \Gamma^{\ k}_{[i\ j]}$

- Covariant derivative of metric is zero. Lagrangian density is proportional to curvature scalar (as in GR).

- Cartan equations:

Torsion is proportional to **spin** density of fermions. ECSK differs significantly from GR at densities > 10^{45} kg/m^3; passes all tests.

$$S_{jik} - S_i g_{jk} + S_k g_{ji} = -\frac{1}{2}\kappa s_{ikj}$$

arXiv.org > gr-qc > arXiv:0911.0334

- Einstein equations:

Curvature is proportional to **energy and momentum** density.

$$G^{ik} = \kappa T^{ik} + \frac{1}{2}\kappa^2\left(s^{ij}{}_{j}s^{kl}{}_{l} - s^{ij}{}_{l}s^{kl}{}_{j} - s^{ijl}s^{k}{}_{jl} + \frac{1}{2}s^{jli}s_{jl}{}^{k} + \frac{1}{4}g^{ik}(2s_{j}{}^{l}{}_{m}s^{jm}{}_{l} - 2s_{j}{}^{l}{}_{l}s^{jm}{}_{m} + s^{jlm}s_{jlm})\right)$$

Universe with spin fluid

Dirac particles can be averaged macroscopically as a spin fluid.

$s^{\mu\nu\rho} = s^{\mu\nu}u^{\rho}$ $\quad s^{\mu\nu}u_{\nu} = 0$ $\quad s^2 = s^{\mu\nu}s_{\mu\nu}/2$

Einstein-Cartan equations for a (closed) FLRW Universe become Friedmann equations for the scale factor *a*.

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa\left(\epsilon - \frac{1}{4}\kappa s^2\right)a^2$$
$$\frac{\dot{a}^2 + 2a\ddot{a}}{c^2} + 1 = -\kappa\left(p - \frac{1}{4}\kappa s^2\right)a^2$$

$$s^2 = \frac{1}{8}(\hbar c n)^2$$

Spin and torsion modify the energy density and pressure with a negative term proportional to the square of the fermion number density *n*, which acts like repulsive gravity.

Universe with spin fluid

For relativistic matter in thermodynamic equilibrium, Friedmann equations can be written in terms of temperature:
$\varepsilon \approx 3p \sim T^4$, $n \sim T^3$.

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa(h_\star T^4 - \alpha h_{nf}^2 T^6)a^2$$

$$\frac{\dot{a}}{a} + \frac{\dot{T}}{T} = 0 \qquad \alpha = \kappa(\hbar c)^2/32$$

$$x = \frac{T}{T_{\rm cr}} \quad y = \frac{a}{a_{\rm cr}} \quad \tau = \frac{ct}{a_{\rm cr}}$$

$$T_{\rm cr} = \left(\frac{2h_\star}{3\alpha h_{nf}^2}\right)^{1/2} \quad a_{\rm cr} = \frac{9\hbar c}{8\sqrt{2}}\left(\frac{\alpha h_{nf}^4}{h_\star^3}\right)^{1/2}$$

Using nondimensional quantities:

$$\dot{y}^2 + 1 = (3x^4 - 2x^6)y^2$$

$$xy = C > 0$$

NP, ApJ 832, 96 (2016); G. Unger & NP, ApJ 870, 78 (2019)

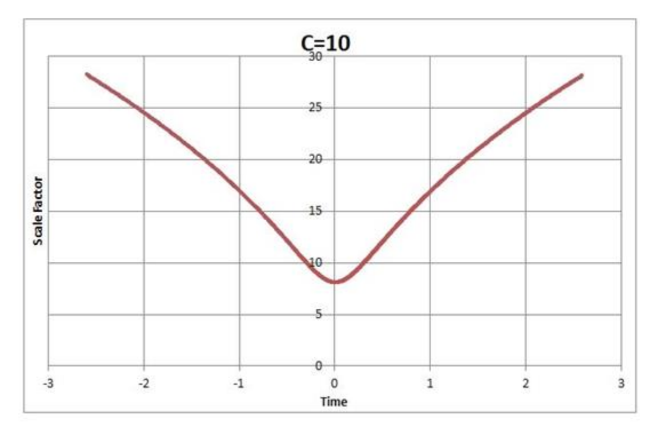
Generating nonsingular bounce

$$\dot{y}^2 + 1 = \frac{3C^4}{y^2} - \frac{2C^6}{y^4}$$

$$y_\pm^2 = 3C^4\left[\frac{1 \pm \sqrt{1 - \frac{8}{9C^2}}}{2}\right]$$

Turning points ($\dot{y} = 0$) for the closed Universe with torsion are positive – **no cosmological singularity!**

- 2 points if $C > (8/9)^{1/2}$ (absolute $y_{min} = 1$ for $C = 1$)
- 1 point if $C = (8/9)^{1/2}$ -> stationary Universe
- 0 points if $C < (8/9)^{1/2}$ -> Universe cannot exist (form)



Matter production causing inflation

Near a bounce, Parker-Starobinsky-Zel'dovich particle production enters through a term ~ H^4, with β as a production parameter.

$$\frac{\dot{a}}{a}\left[1-\frac{3\beta}{c^3 h_{n1} T^3}\left(\frac{\dot{a}}{a}\right)^3\right]=-\frac{\dot{T}}{T}$$

To avoid eternal inflation: the β term < 1 -> $\beta < \beta_{cr} \approx 1/929$.

For $\beta \approx \beta_{cr}$ and during an expansion phase, when $H = \dot{a}/a$ reaches a maximum, the β term is slightly lesser than 1 and:

T ~ const, H ~ const, ε ~ const.

Exponential expansion and mass increase last about t_{Planck}, then H and T decrease. Torsion becomes weak, inflation ends, and radiation dominated era begins. No scalar fields needed.

NP, ApJ 832, 96 (2016)

Einstein-Cartan theory may have consequences in particle physics that can be tested experimentally:

- Fermions are not point particles. PLB 690, 73 (2010)
- Quantum electrodynamics is ultraviolet finite. FOP 50, 900 (2020)

Visitando Machu Picchu, 23 de noviembre de 2018, antes del XII Simposio Latinoamericano de Física de Altas Energías, Pontificia Universidad Católica del Perú, Lima, Perú.

ECSK gravity and spinors

ECSK gravity removes the GR constraint that the affine connection $\Gamma_{i\,j}^{\,k}$ be symmetric by regarding **the antisymmetric part of the connection, the torsion tensor** $S^k_{\ ij} = \Gamma^{\ \ k}_{[i\,j]}$, as a variable. Varying the total Lagrangian density $-\frac{1}{2\kappa}R\sqrt{-g} + L_{\rm m}$, where R is the Ricci scalar and $L_{\rm m}$ is the Lagrangian density of matter, with respect to the contortion tensor $C_{ijk} = S_{ijk} + S_{jki} + S_{kji}$ gives the Cartan equations

$$S^j_{\ ik} - S_i\delta^j_k + S_k\delta^j_i = -\frac{1}{2}\kappa s_{ik}^{\ \ j},$$

where $S_i = S^k_{\ ik}$ and $s^{ijk} = 2(\delta L_{\rm m}/\delta C_{ijk})/\sqrt{-g}$ is the spin tensor.

The Dirac Lagrangian density for a free spinor ψ with mass m, minimally coupled to the gravitational field, is given by $L_{\rm m} = \frac{i}{2}\sqrt{-g}(\bar{\psi}\gamma^i\psi_{;i} - \bar{\psi}_{;i}\gamma^i\psi) - m\sqrt{-g}\bar{\psi}\psi$, where γ^i are the Dirac matrices obeying $\gamma^{(i}\gamma^{k)} = g^{ik}I$, a semicolon denotes a covariant derivative with respect to the connection, and a colon denotes a Riemannian covariant derivative with respect to the Christoffel symbols:

$$\psi_{;k} = \psi_{:k} + \frac{1}{4}C_{ijk}\gamma^{[i}\gamma^{j]}\psi.$$

The resulting spin tensor (and thus torsion) is quadratic in spinor fields:

$$s^{ijk} = -e^{ijkl}s_l, \quad s^i = \frac{1}{2}\bar{\psi}\gamma^i\gamma^5\psi.$$

ECSK gravity and spin fluid

At macroscopic scales, Dirac fields can be averaged and described as a spin fluid:

$$s_{ij}{}^{k} = s_{ij}u^k, \quad s_{ij}u^j = 0.$$

The terms in the effective energy-momentum tensor that are quadratic in the spin tensor do not vanish after averaging:

$$G^{ij} = \kappa\Big(\epsilon - \frac{1}{4}\kappa s^2\Big)u^i u^j - \kappa\Big(p - \frac{1}{4}\kappa s^2\Big)(g^{ij} - u^i u^j),$$

where

$$s^2 = \frac{1}{2}s_{ij}s^{ij} > 0 \propto n_{\mathrm{f}}^2$$

is the averaged square of the spin density.

The Einstein–Cartan equations for a spin fluid are therefore equivalent to the GR Einstein equations for a perfect fluid with:

$$\tilde{\epsilon} = \epsilon - \alpha n_{\mathrm{f}}^2, \quad \tilde{p} = p - \alpha n_{\mathrm{f}}^2,$$

where ϵ and p are the thermodynamic energy density and pressure, n_{f} is the number density of fermions, and $\alpha = \kappa(\hbar c)^2/32$ with $\kappa = 8\pi G/c^4$.

Gravitational collapse of fluid sphere

A spherically symmetric gravitational field is given by the Tolman metric:

$$ds^2 = e^{\nu(\tau,R)}c^2d\tau^2 - e^{\lambda(\tau,R)}dR^2 - e^{\mu(\tau,R)}(d\theta^2 + \sin^2\theta\, d\phi^2),$$

where ν, λ, and μ are functions of a time coordinate τ and a radial coordinate R. Coordinate transformations $\tau \rightarrow \tilde{\tau}(\tau)$ and $R \rightarrow \tilde{R}(R)$ do not change the form of the metric.

The components of the Einstein tensor corresponding to this metric that do not vanish identically are:

$$G_0^0 = -e^{-\lambda}\left(\mu'' + \frac{3\mu'^2}{4} - \frac{\mu'\lambda'}{2}\right) + \frac{e^{-\nu}}{2}\left(\dot{\lambda}\dot{\mu} + \frac{\dot{\mu}^2}{2}\right) + e^{-\mu},$$

$$G_1^1 = -\frac{e^{-\lambda}}{2}\left(\frac{\mu'^2}{2} + \mu'\nu'\right) + e^{-\nu}\left(\ddot{\mu} - \frac{\dot{\mu}\dot{\nu}}{2} + \frac{3\dot{\mu}^2}{4}\right) + e^{-\mu},$$

$$G_2^2 = G_3^3 = -\frac{e^{-\nu}}{4}(\dot{\lambda}\dot{\nu} + \dot{\mu}\dot{\nu} - \dot{\lambda}\dot{\mu} - 2\ddot{\lambda} - \dot{\lambda}^2 - 2\ddot{\mu} - \dot{\mu}^2)$$

$$-\frac{e^{-\lambda}}{4}(2\nu'' + \nu'^2 + 2\mu'' + \mu'^2 - \mu'\lambda' - \nu'\lambda' + \mu'\nu'),$$

$$G_0^1 = \frac{e^{-\lambda}}{2}(2\dot{\mu}' + \dot{\mu}\mu' - \dot{\lambda}\mu' - \dot{\mu}\nu'),$$

where a dot denotes differentiation with respect to $c\tau$ and a prime denotes differentiation with respect to R.

Gravitational collapse of fluid sphere

In the comoving frame of reference, the spatial components of the four-velocity u^μ vanish. The nonzero components of the energy–momentum tensor for a spin fluid, $T_{\mu\nu} = (\tilde{\epsilon} + \tilde{p})u_\mu u_\nu - \tilde{p}g_{\mu\nu}$, are: $T^0_0 = \tilde{\epsilon}$, $T^1_1 = T^2_2 = T^3_3 = -\tilde{p}$. The Einstein field equations $G^\mu_\nu = \kappa T^\mu_\nu$ in this frame of reference are:

$$G^0_0 = \kappa\tilde{\epsilon}, \quad G^1_1 = G^2_2 = G^3_3 = -\kappa\tilde{p}, \quad G^1_0 = 0.$$

The covariant conservation of the energy–momentum tensor gives

$$\dot{\lambda} + 2\dot{\mu} = -\frac{2\dot{\tilde{\epsilon}}}{\tilde{\epsilon} + \tilde{p}}, \;\; \nu' = -\frac{2\tilde{p}'}{\tilde{\epsilon} + \tilde{p}},$$

where the constants of integration depend on the allowed transformations $\tau \to \tilde{\tau}$ and $R \to \tilde{R}$.

If the pressure is homogeneous (no pressure gradients), then $\tilde{p}' = 0$, which gives $\nu' = 0$. Therefore, $\nu = \nu(\tau)$ and a transformation $\tau \to \tilde{\tau}$ can bring ν to zero and $g_{00} = e^\nu$ to 1. The system of coordinates becomes synchronous. Defining $r(\tau, R) = e^{\mu/2}$ turns the metric into

$$ds^2 = c^2 d\tau^2 - e^{\lambda(\tau,R)}dR^2 - r^2(\tau, R)(d\theta^2 + \sin^2\theta\, d\phi^2).$$

Every particle in a collapsing fluid sphere is represented by a radial coordinate R that ranges from 0 (at the center) to R_0 (at the surface).

Gravitational collapse of fluid sphere

The Einstein field equations reduce to

$$
\begin{aligned}
&\kappa\tilde{\epsilon} = -\frac{e^{-\lambda}}{r^2}(2rr'' + r'^2 - rr'\lambda') + \frac{1}{r^2}(r\dot{r}\dot{\lambda} + \dot{r}^2 + 1),\\
&-\kappa\tilde{p} = \frac{1}{r^2}(-e^{-\lambda}r'^2 + 2r\ddot{r} + \dot{r}^2 + 1),\\
&-2\kappa\tilde{p} = -\frac{e^{-\lambda}}{r}(2r'' - r'\lambda') + \frac{\dot{r}\dot{\lambda}}{r} + \ddot{\lambda} + \frac{1}{2}\dot{\lambda}^2 + \frac{2\ddot{r}}{r},\\
&2\dot{r}' - \dot{\lambda}r' = 0.
\end{aligned}
$$

Integrating the last equation gives

$$
e^{\lambda} = \frac{r'^2}{1 + f(R)}, \tag{1}
$$

where f is a function of R satisfying a condition $1 + f > 0$ (Landau & Lifshitz, *The Classical Theory of Fields*). Substituting this relation into the second field equation gives $2r\ddot{r} + \dot{r}^2 - f = -\kappa\tilde{p}r^2$, which is integrated to

$$
\dot{r}^2 = f(R) + \frac{F(R)}{r} - \frac{\kappa}{r}\int \tilde{p}r^2 dr,
$$

where F is a positive function of R.

Gravitational collapse of fluid sphere

Substituting the last two equations into the first field equation gives a relation $\kappa(\tilde{\epsilon}+\tilde{p}) = F'(R)/(r^2 r')$, leading to

$$\dot{r}^2 = f(R) + \frac{\kappa}{r}\int_0^R \tilde{\epsilon} r^2 r' dR. \tag{2}$$

If the mass of the sphere is M, then the Schwarzschild radius $r_g = 2GM/c^2$ of the black hole that forms from the sphere is equal to

$$r_g = \kappa \int_0^{R_0} \tilde{\epsilon} r^2 r' dR.$$

These two quations give $\dot{r}^2(\tau, R_0) = f(R_0) + r_g/r(\tau, R_0)$. If $r_0 = r(0, R_0)$ is the initial radius of the sphere and the sphere is initially at rest, then $\dot{r}(0, R_0) = 0$. Consequently, the value of R_0 is determined by

$$f(R_0) = -\frac{r_g}{r_0}.$$

Substituting $r = e^{\mu/2}$ and (1) into the first conservation law gives the first law of thermodynamics for the effective energy density and pressure:

$$\frac{d}{d\tau}(\tilde{\epsilon} r^2 r') + \tilde{p}\frac{d}{d\tau}(r^2 r') = 0. \tag{3}$$

Collapse of spin fluid sphere

If we assume that the spin fluid is composed of an ultrarelativistic matter in kinetic equilibrium, then $\epsilon = h_\star T^4$, $p = \epsilon/3$, and $n_\mathrm{f} = h_{n\mathrm{f}} T^3$, where T is the temperature of the fluid, $h_\star = (\pi^2/30)(g_\mathrm{b} + (7/8)g_\mathrm{f})k^4/(\hbar c)^3$, and $h_{n\mathrm{f}} = (\zeta(3)/\pi^2)(3/4)g_\mathrm{f}k^3/(\hbar c)^3$. For standard-model particles, $g_\mathrm{b} = 29$ and $g_\mathrm{f} = 90$. The effective energy density and pressure are thus:

$$\tilde{\epsilon} = h_\star T^4 - \alpha h_{n\mathrm{f}}^2 T^6, \quad \tilde{p} = \frac{1}{3} h_\star T^4 - \alpha h_{n\mathrm{f}}^2 T^6.$$

Since the pressure has no gradient, the temperature only depends on τ, and so does the energy density. This scenario describes a homogeneous sphere. The first law of thermodynamics (3) gives

$$r^2 r' T^3 = g(R), \tag{4}$$

where g is a function of R. Putting this relation into (2) gives

$$\dot{r}^2 = f(R) + \frac{\kappa}{r}(h_\star T^4 - \alpha h_{n\mathrm{f}}^2 T^6) \int_0^R r^2 r' dR. \tag{5}$$

Equations (4) and (5) give the function $r(\tau, R)$, which with (1) gives $\lambda(\tau, R)$. The integration of (5) also contains the initial value $\tau_0(R)$. The metric therefore depends on three arbitrary functions: $f(R)$, $g(R)$, and $\tau_0(R)$.

N. Popławski, arXiv:2008.02136.

Collapse of spin fluid sphere

We seek a solution of (4) and (5) as

$$f(R) = -\sin^2 R, \quad r(\tau, R) = a(\tau)\sin R, \tag{6}$$

where $a(\tau)$ is a nonnegative function of τ. For this choice of functions, (4) gives $a^3T^3\sin^2 R\cos R = g(R)$, in which separation of the variables τ and R leads to

$$g(R) = \mathrm{const}\cdot\sin^2 R\cos R, \quad a^3T^3 = \mathrm{const}.$$

Consequently,

$$aT = a_0T_0, \quad \frac{\dot{T}}{T} + \frac{H}{c} = 0, \tag{7}$$

where $a_0 = a(\tau = 0)$ and $T_0 = T(\tau = 0)$ are the values at the initial time, and $H = c\dot{a}/a$. Substituting (6) into (5) gives

$$\dot{a}^2 + 1 = \frac{\kappa}{3}(h_\star T^4 - \alpha h_{nf}^2 T^6)a^2, \tag{8}$$

which has a form of the Friedmann equation for the scale factor a as a function of the cosmic time τ in a closed, homogeneous universe. The quantity H is the Hubble parameter of this universe. Using (7) in (8) yields

$$\dot{a}^2 = -1 + \frac{\kappa}{3}\Big(\frac{h_\star T_0^4 a_0^4}{a^2} - \frac{\alpha h_{nf}^2 T_0^6 a_0^6}{a^4}\Big). \tag{9}$$

Collapse of spin fluid sphere

The relations (6) determine the constants:

$$\sin R_0 = \left(\frac{r_g}{r_0}\right)^{1/2}, \quad a(0) = \left(\frac{r_0^3}{r_g}\right)^{1/2}.$$

Substituting the initial values $a(0)$ and $\dot{a}(0) = 0$ into (8), in which the second term on the right-hand side is negligible, gives $Mc^2 = (4\pi/3)r_0^3 h_\star T_0^4$. This relation indicates the equivalence of mass and energy of a fluid sphere with radius r_0 and determines T_0. An event horizon for the entire sphere forms when $r(\tau, R_0) = r_g$, which is equivalent to $a = (r_g r_0)^{1/2}$. Equation (9) has two turning points, $\dot{a} = 0$, if

$$\frac{r_0^3}{r_g} > \frac{3\pi G\hbar^4 h_{nf}^4}{8h_\star^3} \sim l_{\mathrm{Planck}}^2,$$

which is satisfied for astrophysical systems that form black holes.

Substituting (6) into (1) gives $e^{\lambda(\tau,R)} = a^2$. Consequently, the square of an infinitesimal interval in the interior of a collapsing spin fluid is given by

$$ds^2 = c^2 d\tau^2 - a^2(\tau)dR^2 - a^2(\tau)\sin^2 R(d\theta^2 + \sin^2\theta\, d\phi^2).$$

This metric has a form of the closed Friedmann–Lemaître–Robertson–Walker metric and describes a part of a **closed universe** with $0 \leq R \leq R_0$ (like dust).

Nonsingular bounce

Equation (9) can be solved analytically in terms of an elliptic integral of the second kind, giving the function $a(\tau)$ and then $r(\tau, R) = a(\tau)\sin R$:

$$\dot{a}^2 = -1 + \frac{\kappa}{3}\Big(\frac{h_\star T_0^4 a_0^4}{a^2} - \frac{\alpha h_{nf}^2 T_0^6 a_0^6}{a^4}\Big). \tag{9}$$

The **value of a never reaches zero** because as a decreases, the right-hand side of (9) becomes negative, contradicting the left-hand side. The change of the sign occurs when $a < (r_g r_0)^{1/2}$, that is, after the event horizon forms. Consequently, all particles with $R > 0$ fall within the event horizon but never reach $r = 0$ (the only particle at the center is the particle that is initially at the center, with $R = 0$). **A singularity is replaced with a nonsingular bounce.** Nonzero values of a give finite values of T and thus finite values of ϵ, p, and $n_{\rm f}$.

After the bounce, the matter expands on the other side of the event horizon as a **new universe**. This universe has a closed geometry (constant positive curvature). The quantity $a(\tau)$ is the scale factor. The universe is oscillatory: the value of a oscillates between the two turning points. The value of R_0 does not change. A turning point at which $\ddot{a} > 0$ is a bounce, and a turning point at which $\ddot{a} < 0$ is a crunch. This universe has an infinite number of identical cycles.

N. Popławski, Astrophys. J. 832, 96 (2016).
G. Unger and N. Popławski, Astrophys. J. 870, 78 (2019).

Nonsingular bounce

The Raychaudhuri equation for a congruence of geodesics without four-acceleration and rotation is $d\theta/ds = -\theta^2/3 - 2\sigma^2 - R_{\mu\nu}u^\mu u^\nu$, where θ is the expansion scalar, σ^2 is the shear scalar, and $R_{\mu\nu}$ is the Ricci tensor. For a spin fluid, the last term in this equation is equal to $-\kappa(\tilde{\epsilon}+3\tilde{p})/2$. Consequently, the necessary and sufficient condition for avoiding a singularity in a black hole is $-\kappa(\tilde{\epsilon}+3\tilde{p})/2 > 2\sigma^2$. For a relativistic spin fluid, $p=\epsilon/3$, this condition is equivalent to

$$2\kappa\alpha n_{\mathrm{f}}^2 > 2\sigma^2 + \kappa\epsilon. \tag{10}$$

Without torsion, the left-hand side of (10) would be absent and this inequality could not be satisfied, resulting in a singularity. **Torsion may provide a necessary condition for preventing a singularity.** In the absence of shear, this condition would be also sufficient.
(Hehl, Trautman, Kopczyński, Tafel, Kuchowicz)

The presence of shear opposes the effects of torsion. The shear scalar σ^2 grows with decreasing a like $\sim a^{-6}$, which is the same power law as that for n_{f}^2. Therefore, if the initial shear term dominates over the initial torsion term in (10), then it will dominate at later times during contraction and a singularity will form. To avoid a singularity if the shear is present, n_{f}^2 must grow faster than $\sim a^{-6}$. Consequently, **fermions must be produced** in a black hole during contraction.

Nonsingular bounce

The production rate of particles in a contracting or expanding universe can be phenomenologically given by

$$\frac{1}{c\sqrt{-g}}\frac{d(\sqrt{-g}n_{\mathrm{f}})}{dt} = \frac{\beta H^4}{c^4}, \qquad (11)$$

where $g = -a^6 \sin^4 R \sin^2\theta$ is the determinant of the metric tensor and β is a nondimensional production rate. With particle production, the second equation in (7) turns into

$$\frac{\dot{T}}{T} = \frac{H}{c}\Big(\frac{\beta H^3}{3c^3 h_{n\mathrm{f}} T^3} - 1\Big). \qquad (12)$$

Particle production changes the power law $n_{\mathrm{f}}(a)$:

$$n_{\mathrm{f}} \sim a^{-(3+\delta)},$$

where δ varies with τ. Putting this relation into (11) gives

$$\delta \sim -a^{\delta}\dot{a}^3.$$

During contraction, $\dot{a} < 0$ and thus $\delta > 0$. The term $n_{\mathrm{f}}^2 \sim a^{-6-2\delta}$ grows faster than $\sigma^2 \sim a^{-6}$ and a singularity is avoided. **Particle production and torsion together reverse the effects of shear, generating a bounce.**

The universe in a black hole

The dynamics of the nonsingular, relativistic universe in a black hole is described by equations (8) and (12):

$$\dot{a}^2 + 1 = \frac{\kappa}{3}(h_\star T^4 - \alpha h_{nf}^2 T^6)a^2, \quad \frac{\dot{T}}{T} = \frac{H}{c}\Big(\frac{\beta H^3}{3c^3 h_{nf} T^3} - 1\Big),$$

where $H = c\dot{a}/a$. These equations, with the initial conditions $a(0) = (r_0^3/r_g)^{1/2}$ and $\dot{a}(0) = 0$, give the functions $a(\tau)$ and $T(\tau)$.

The shear would enter the right-hand side of the first equation as an additional positive term that is proportional to a^{-4}. When the universe becomes nonrelativistic, the term $h_\star T^4$ changes into a positive term proportional to a^{-1}. The cosmological constant enters as a positive term proportional to a^2.

Particle production increases the maximum size of the scale factor that is reached at a crunch. Consequently, a new cycle is larger and lasts longer then the previous cycle. R_0 is given by $\sin^3 R_0 = r_g/a(0)$, where $a(0)$ is the maximum scale factor in the first cycle. Since the maximum scale factor in the next cycle is larger, the value of $\sin R_0$ decreases. As cycles proceed, R_0 approaches π (the value for a completely closed universe).

A parent black hole creating a new, baby universe becomes an **Einstein–Rosen bridge** (unidirectional wormhole) to that universe.

Inflation

During expansion ($H > 0$), if β is too big, then the right-hand side could become positive:

$$\frac{\dot{T}}{T} = \frac{H}{c}\Big(\frac{\beta H^3}{3c^3 h_{n\mathrm{f}} T^3} - 1\Big).$$

In this case, the temperature would grow with increasing a, which would lead to eternal inflation. Consequently, there is an upper limit to the production rate: the maximum of the function $(\beta H^3)/(3c^3 h_{n\mathrm{f}} T^3)$ must be lesser than 1.

If $(\beta H^3)/(3c^3 h_{n\mathrm{f}} T^3)$ increases after a bounce to a value that is slightly lesser than 1, then T would become approximately constant. Accordingly, H would be also nearly constant and the scale factor a would grow exponentially, generating inflation. Since the energy density would be also nearly constant, the universe would produce enormous amounts of matter and entropy. Such an expansion would last until the right-hand side of drops below 1. Consequently, inflation would last a finite period of time. After this period, the effects of torsion weaken and the universe smoothly enters the radiation-dominated expansion, followed by the matter-dominated expansion.

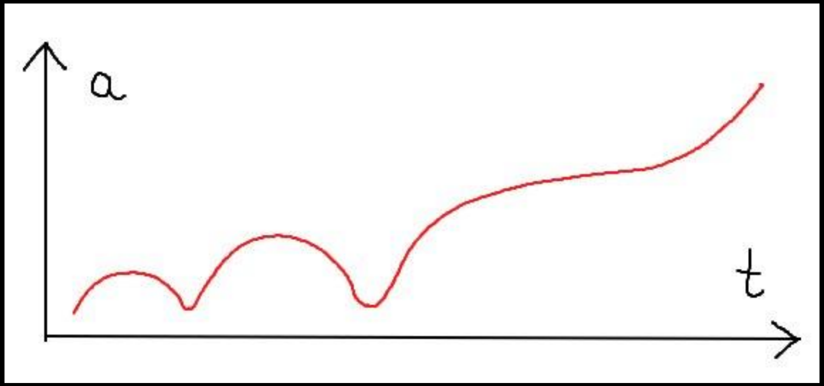
Torsion and particle production can generate **finite inflation** without scalar fields and reheating.

N. J. Popławski, Phys. Lett. B 694, 181 (2010).

N. Popławski, Astrophys. J. 832, 96 (2016).

Dark energy ends oscillations

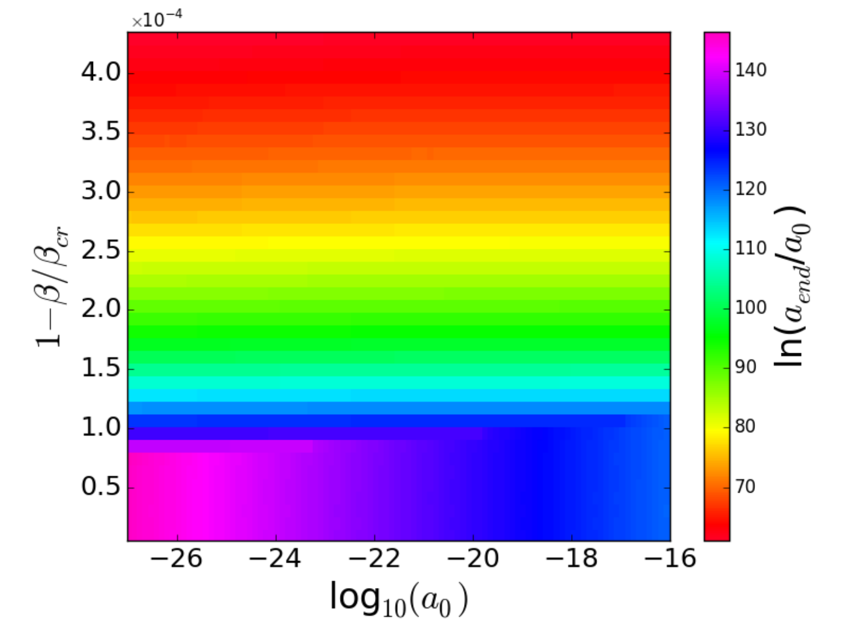
If quantum effects in the gravitational field near a bounce do not produce enough matter, then the closed Universe reaches the maximum size and then contracts to another bounce, beginning the new cycle. Because of matter production, a new cycle reaches larger size and last longer than the previous cycle.



When the Universe reaches a size at which the cosmological constant is dominating, then it avoids another contraction and starts expanding to infinity.

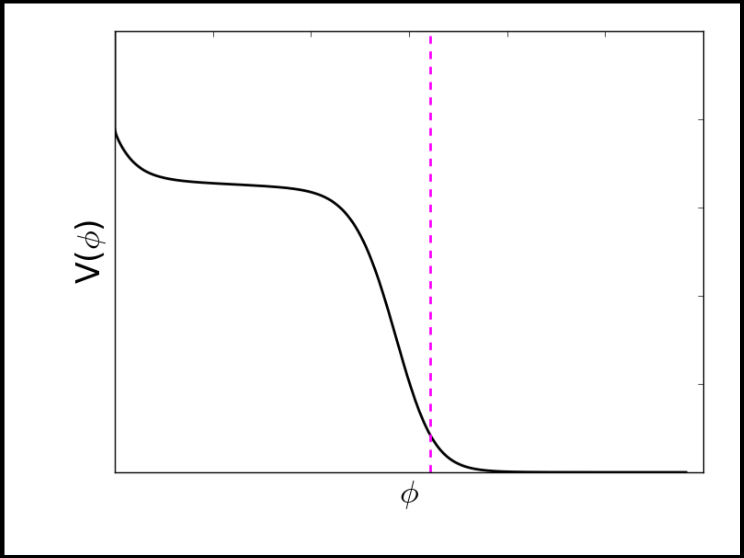
- The numbers of e-folds and bounces (until the Universe reaches the radiation-matter equality) depend on the particle production but are not too sensitive to the initial scale factor.
- The Big Bang was the last bounce (Big Bounce).

β/β_{cr}	Number of bounces
0.996	1
0.984	2
0.965	3
0.914	5
0.757	10



S. Desai & NP, PLB 755, 183 (2016)

It is possible to find a scalar field potential which generates the same time dependence of the scale factor.



$\beta/\beta_{cr} = 0.9998$

Plateau-like (slow-roll) potential – favored by Planck satellite data.

Torsion reproduces the same shape without hypothetical scalar fields and with only 1 parameter: particle production rate.

It predicts the CMB parameters consistent with the data.

Every black hole creates a new universe? Our Universe originated in a black hole?

Every black hole may create a new, closed, baby universe (Novikov, Pathria, Hawking, Smolin, NP).

Accordingly, our Universe may be closed and may have born in the interior of a black hole existing in a parent universe.
NP, PLB 694, 181 (2010)

This hypothesis should solve the black hole information paradox: the information goes through the Einstein-Rosen bridge to the baby universe on the other side of the black hole's event horizon.

The motion through an event horizon is one way only: it defines the past and future. Time asymmetry at the event horizon may induce time asymmetry everywhere in the baby universe and explain why time flows in one direction.

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References:

Kibble, JMP 2, 212 (1961); Sciama, RMP 36, 463 (1964)
Hehl, von der Heyde & Kerlick, PRD 10, 1066 (1974)
Hehl, von der Heyde, Kerlick & Nester, RMP 48, 393 (1976)
Ellis & Madsen, CQG 8, 667 (1991)
Nomura, Shirafuji & Hayashi, PTP 86, 1239 (1991)
NP, arXiv:0911.0334; PLB 690, 73 (2010)
NP, PLB 687, 110 (2010); PLB 694, 181 (2010); GRG 44, 1007 (2012); PRD 85, 107502 (2012); Unger & NP, ApJ 870, 78 (2019); Cubero & NP, CQG 37, 025011 (2020)
Desai & NP, PLB 755, 183 (2016)
NP, ApJ 832, 96 (2016); IJMPD 27, 1847020 (2018); MPLA 33, 1850236 (2018)
NP, JETP (2020), arXiv:2008.02136

Resumen: el Universo en un agujero negro

- La ley de conservación del momento angular total en el espacio-tiempo curvo, consistente con la ecuación de Dirac, requiere torsión. La teoría más simple con torsión, la gravedad de Einstein-Cartan, tiene el mismo Lagrangiano que RG, pero la conexión afín contiene el tensor de torsión, generado por el espín.
- El colapso gravitacional de una esfera esféricamente simétrica de un fluido de espín crea un horizonte de eventos. La materia dentro del horizonte se colapsa a densidades extremadamente altas, en las que la torsión actúa como repulsión gravitacional.
- Sin cizallamiento, la torsión evita una singularidad y la reemplaza por un rebote no singular. Con el cizallamiento, la torsión evita una singularidad si el número de fermiones aumenta durante la contracción a través de la producción de partículas cuánticas.
- La producción de partículas durante la expansión produce enormes cantidades de materia y puede generar un período finito de inflación. El universo cerrado resultante en el otro lado del horizonte de eventos puede tener varios rebotes. Tal universo es oscilatorio, con cada ciclo de mayor tamaño que el ciclo anterior, hasta que alcanza el tamaño cosmológico y se expande indefinidamente.

Gracias

Problemas de la relatividad general

La cosmología se basa en la relatividad general (RG), que describe la gravedad como la curvatura del espacio-tiempo.

- Singularidades: puntos con densidad de materia infinita.
- Incompatible con la mecánica cuántica. La gravedad cuántica puede resolver el problema de la singularidad.
- Las ecuaciones de campo contienen la conservación del momento angular orbital, lo que contradice la ecuación de Dirac que proporciona la conservación del momento angular total (orbital + espín) y permite el intercambio espín-órbita en mecánica cuántica.

La extensión clásica más simple de RG para incluir el espín: **Teoría de Einstein-Cartan**. Puede resolver el problema de la singularidad.

Problemas de la cosmología

- Singularidad del Big-Bang.
- ¿Qué causó el Big Bang? ¿Qué existía antes?
- La inflación (expansión exponencial del Universo temprano) resuelve los problemas de planitud y horizonte, y predice el espectro observado de perturbaciones de CMB. ¿Qué causó la inflación? (se suelen utilizar campos escalares hipotéticos)
- ¿Cómo terminó la inflación? (sin inflación eterna)

La teoría de Einstein-Cartan reemplaza el Big Bang por un no singular **Gran Rebote** (Big Bounce). La dinámica después del rebote puede explicar los problemas de planitud y horizonte.

Gravedad de Einstein-Cartan-Sciama-Kibble

- El **tensor de torsión** es una variable además de la métrica. Es la parte antisimétrica de la conexión afín.

$$S^k{}_{ij} = \Gamma^{\ k}_{[i\ j]}$$

- La derivada covariante de la métrica desaparece (como en RG), y la conexión tiene una parte puramente métrica y una parte de torsión.

- La densidad Lagrangiana es proporcional al escalar de curvatura de Ricci R (como en RG), construido a partir de la conexión. El tensor de curvatura se puede descomponer en una puramente métrica parte y parte que contiene la torsión y sus derivadas.

arXiv.org > gr-qc > arXiv:0911.0334

Hehl, von der Heyde, Kerlick & Nester, RMP 48, 393 (1976)

Gravedad de Einstein-Cartan-Sciama-Kibble

- La variación de la acción total de la gravedad y la materia con respecto a la torsión da las ecuaciones de campo de Cartan:

$$S_{jik} - S_i g_{jk} + S_k g_{ji} = -\frac{1}{2}\kappa s_{ikj}$$

La **torsión** es proporcional a la densidad de **espín** de los fermiones. EC difiere significativamente del RG en densidades > 10^{45} kg/m^3. En vacío, se reduce a RG. EC pasa todas las pruebas que hace RG.

- La variación con respecto a la métrica da las ecuaciones de Einstein: la **curvatura** es proporcional a la densidad de **energía y momento**. Usando la descomposición por curvatura, se pueden escribir como RG con el tensor de energía-momento con términos cuadráticos en densidad de espín.

$$G^{ik} = \kappa T^{ik} + \frac{1}{2}\kappa^2\left(s^{ij}{}_j s^{kl}{}_l - s^{ij}{}_l s^{kl}{}_j - s^{ijl} s^k{}_{jl} + \frac{1}{2} s^{jli} s_{jl}{}^k + \frac{1}{4} g^{ik}(2 s_j{}^l{}_m s^{jm}{}_l - 2 s_j{}^l{}_l s^{jm}{}_m + s^{jlm} s_{jlm})\right)$$