Big Bounce and Dark Energy in Gravity with Torsion

Nikodem J. Popławski

Lecture
Virtual Institute of Astroparticle Physics
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Outline

1. Problems of standard cosmology
2. Einstein-Cartan-Sciama-Kibble theory of gravity
3. Dirac spinors in spacetime with torsion
4. Solution: cosmology with torsion
    - Nonsingular big bounce instead of singular big bang
    - Torsion as simplest alternative to inflation
5. Simplest affine theory of gravity
    - Cosmological constant from torsion

Problems of standard cosmology

- **Big-bang singularity** – can be solved by LQG

But LQG has not been shown to reproduce GR in classical limit

- Flatness and horizon problems – solved by inflation

consistent with cosmological perturbations observed in CMB

But:

- Scalar field with a specific (slow-roll) potential needed

fine-tuning problem not resolved

- What physical field causes inflation?
- What ends inflation?

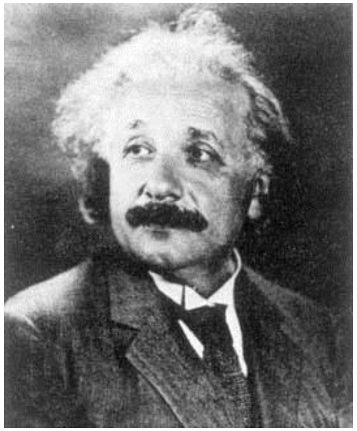
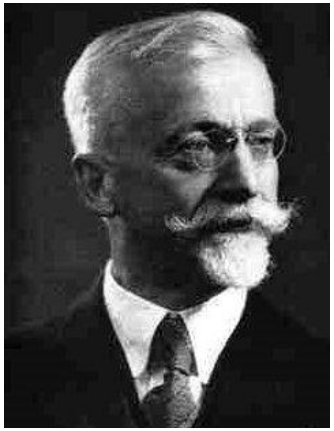
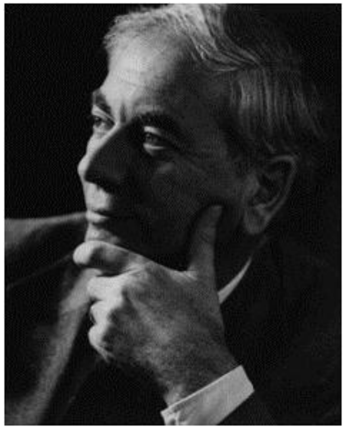
Existing alternatives to GR:

- Use exotic fields
- Are more complicated
- Do not address all problems (usually 1, sometimes 2)

- Dark energy
- Dark matter
- Matter-antimatter asymmetry

Einstein-Cartan-Sciama-Kibble theory

Spacetime with gravitational **torsion**

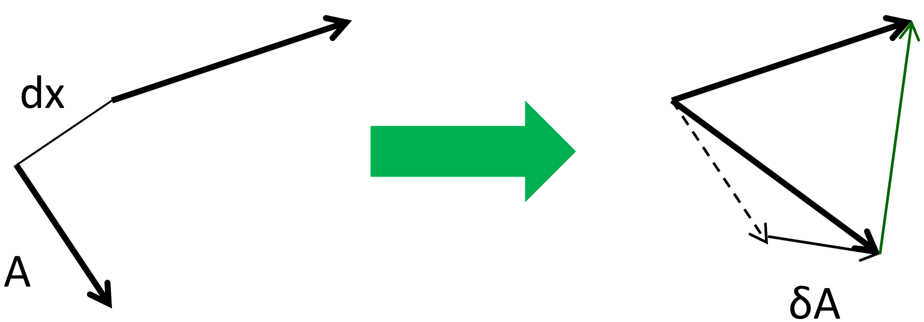
This talk:
Big-bang singularity, inflation and dark energy problems all naturally solved by **torsion**

Affine connection

- Vectors & tensors – under coordinate transformations behave like differentials and gradients & their products.

- Differentiation of vectors in curved spacetime requires subtracting two vectors at two infinitesimally separated points with different transformation properties.

- **Parallel transport** brings one vector to the origin of the other, so that their difference would make sense.



$$\delta A^k = -\Gamma_{l\,i}^{\,k} A^l dx^i$$

Affine connection

Curvature and torsion

Calculus in curved spacetime requires geometrical structure: affine connection

Covariant derivative of a vector

$$B^k{}_{;i} = B^k{}_{,i} + \Gamma_l{}^k{}_i B^l$$

Two tensors constructed from affine connection:

- Curvature tensor

$$R^i{}_{mjk} = \partial_j \Gamma_m{}^i{}_k - \partial_k \Gamma_m{}^i{}_j + \Gamma_l{}^i{}_j \Gamma_m{}^l{}_k - \Gamma_l{}^i{}_k \Gamma_m{}^l{}_j$$

- **Torsion tensor** – antisymmetric part of connection

$$S^k{}_{ij} = \Gamma^{\ k}_{[i\ j]}$$

É. Cartan (1921)

Contortion tensor $C^i{}_{jk} = S^i{}_{jk} + S_{jk}{}^i + S_{kj}{}^i$

Theories of spacetime

Special Relativity – flat spacetime (no affine connection)
Dynamical variables: matter fields

General Relativity – (curvature, no torsion)
Dynamical variables: matter fields + metric tensor g_{ik}

$$S^k{}_{ij} = 0$$

Connection restricted to be symmetric – ad hoc (equivalence principle)

Degrees of freedom

ECSK gravity (simplest theory with curvature & torsion)
Dynamical variables: matter fields + metric + **torsion**

ECSK gravity

T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961)
D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964)

Riemann-Cartan spacetime – metricity $g_{ik;j} = 0$

$\rightarrow \Gamma_{i\,j}^{\,k} = \{^{\,k}_{i\,j}\} + C^k{}_{ij}$

Christoffel symbols of metric

Matter Lagrangian density

Total Lagrangian density like in GR: $-\frac{1}{2\kappa}R\sqrt{-g} + \mathfrak{L}_m$

Two tensors describing matter:

- Energy-momentum tensor $T_{ik} = 2(\delta\mathfrak{L}_m/\delta g^{ik})/\sqrt{-g}$
- **Spin tensor** $s^{ijk} = 2(\delta\mathfrak{L}_m/\delta C_{ijk})/\sqrt{-g}$

ECSK gravity

Curvature tensor = Riemann tensor
+ tensor quadratic in torsion + total derivative

Stationarity of action under δg^{ik} → **Einstein equations**

$$G_{ik} = \kappa(T_{ik} + U_{ik})$$

$$U_{ik} = \frac{1}{\kappa}\Big(C^j{}_{ij}C^l{}_{kl} - C^l{}_{ij}C^j{}_{kl} - \frac{1}{2}g_{ik}(C^{jm}{}_j C^l{}_{ml} - C^{mjl}C_{ljm})\Big)$$

Stationarity of action under δC_{ijk} → **Cartan equations**

$$S^j{}_{ik} - S_i\delta^j_k + S_k\delta^j_i = -\frac{1}{2}\kappa s_{ik}{}^j \qquad S_i = S^k{}_{ik}$$

- Torsion is **proportional** to spin density
- Contributions to energy-momentum from spin are **quadratic**

Dirac spinors with torsion

Simplest case: minimal coupling

$$\gamma^{(i}\gamma^{k)} = g^{ik} I$$

Dirac Lagrangian density (natural units)

$$\mathfrak{L}_m = \frac{i}{2}\sqrt{-g}(\bar{\psi}\gamma^i\psi_{;i} - \bar{\psi}_{;i}\gamma^i\psi) - m\sqrt{-g}\,\bar{\psi}\psi$$

Dirac equation

$$i\gamma^k\psi_{;k} = m\psi$$

$$\psi_{;k} = \psi_{:k} + \frac{1}{4}C_{ijk}\gamma^{[i}\gamma^{j]}\psi$$

$$\bar{\psi}_{;k} = \bar{\psi}_{:k} - \frac{1}{4}C_{ijk}\bar{\psi}\gamma^{[i}\gamma^{j]}$$

Covariant derivative of a spinor

GR covariant derivative of a spinor

arXiv.org > gr-qc > arXiv:0911.0334

F. W. Hehl, P. von der Heyde, G. D. Kerlick & J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976)

Dirac spinors with torsion

Spin tensor is completely antisymmetric

$$s^{ijk} = -e^{ijkl} s_l \qquad s^i = \frac{1}{2}\bar{\psi}\gamma^i\gamma^5\psi$$

Torsion and contortion tensors are also antisymmetric

$$C_{ijk} = S_{ijk} = \frac{1}{2}\kappa e_{ijkl} s^l$$

LHS of Einstein equations

$$T_{ik} + U_{ik} = \frac{i}{2}(\bar{\psi}\delta^j_{(i}\gamma_{k)}\psi_{:j} - \bar{\psi}_{:j}\delta^j_{(i}\gamma_{k)}\psi) + \frac{3}{4}\kappa s^l s_l g_{ik}$$

comoving frame

$$-\tfrac{3}{4}\kappa \mathbf{s}^2 g_{ik}$$

$$\langle \mathbf{s}^2 \rangle = \tfrac{3}{4} n^2$$

Fermion number density

ECSK gravity

Torsion significant when $U_{ik} \sim T_{ik}$ (at Cartan density)

$$\rho_{\mathrm{C}} = \frac{m_{\mathrm{n}}^2 c^4}{G\hbar^2}$$

For fermionic matter ρ_C > 10^{45} kg m^{-3} >> nuclear density

Other existing fields do not generate torsion

- Gravitational effects of torsion are negligible even for neutron stars (ECSK passes all tests of GR)
- Torsion vanishes in vacuum → ECSK reduces to GR
- Torsion is significant in **very early Universe** and **black holes**

Imposing symmetric connection is unnecessary

ECSK has less assumptions than GR

Cosmology with torsion

Spin corrections to energy-momentum act like a perfect fluid

$$\tilde{\epsilon} = -\tilde{p} = -\alpha n^2 \qquad \alpha = \frac{9}{16}\kappa$$

Friedman equations for a homogeneous and isotropic Universe:

$$\dot{a}^2 + k = \frac{1}{3}\kappa(\epsilon - \alpha n^2)a^2$$

$$a^3 d\epsilon - 2\alpha a^3 n dn + (\epsilon + p)d(a^3) = 0$$

Statistical physics in early Universe (neglect *k*)

$$\epsilon(T) = \underbrace{\tfrac{\pi^2}{30} g_\star}_{h_\star}(T)T^4 \qquad p(T) = \frac{\epsilon(T)}{3} \qquad n(T) = \underbrace{\tfrac{\zeta(3)}{\pi^2} g_n}_{h_n}(T)T^3$$

Cosmology with torsion

NP, Phys. Rev. D **85**, 107502 (2012)

Scale factor vs. temperature

$$\frac{dT}{T} - \frac{3\alpha h_n^2}{2h_\star} T dT + \frac{da}{a} = 0$$

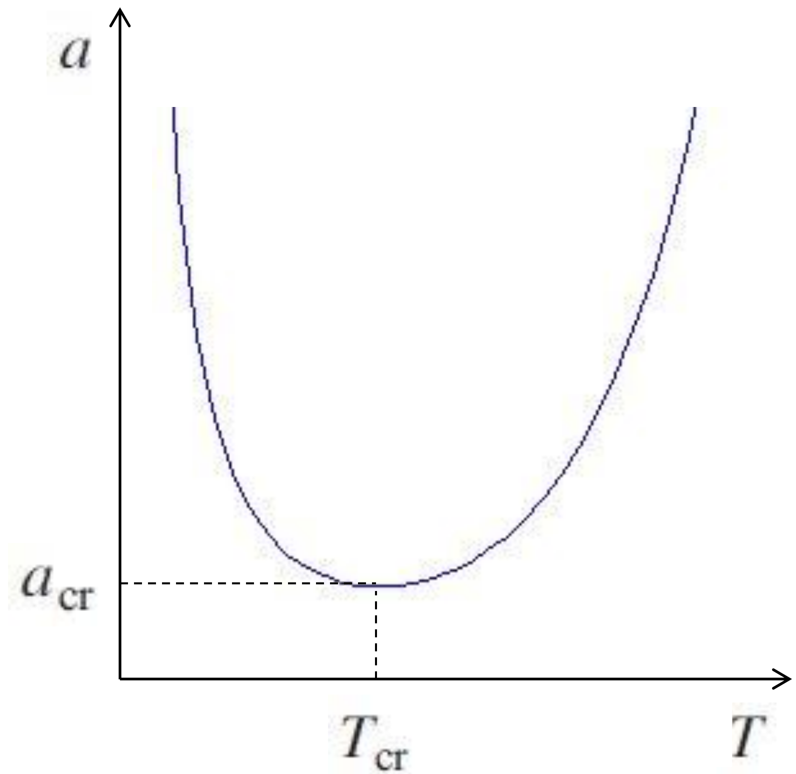
Solution

reference values

$$a = \frac{a_r T_r}{T} \underbrace{\exp\left(\frac{3\alpha h_n^2}{4h_\star} T^2\right)}_{\text{torsion correction}}$$

$$a \geq a_{\mathrm{cr}}$$

Singularity avoided



$$T_{\mathrm{cr}} = \left(\frac{2h_\star}{3\alpha h_n^2}\right)^{1/2}$$

Cosmology with torsion

Temperature vs. time

$$\dot{T}^2\left(\frac{1}{T^2} - \frac{3\alpha h_n^2}{2h_\star}\right)^2 = \frac{\kappa}{3}(h_\star T^2 - \alpha h_n^2 T^4)$$

$$|\dot{\beta}| = \sqrt{\frac{\kappa h_\star}{3}} \frac{\sqrt{\beta^2 - \frac{2}{3}\beta_{cr}^2}}{\beta^2 - \beta_{cr}^2} \qquad \beta = T^{-1} \qquad \rightarrow \quad T \leq T_{cr}$$

Can be integrated parametrically

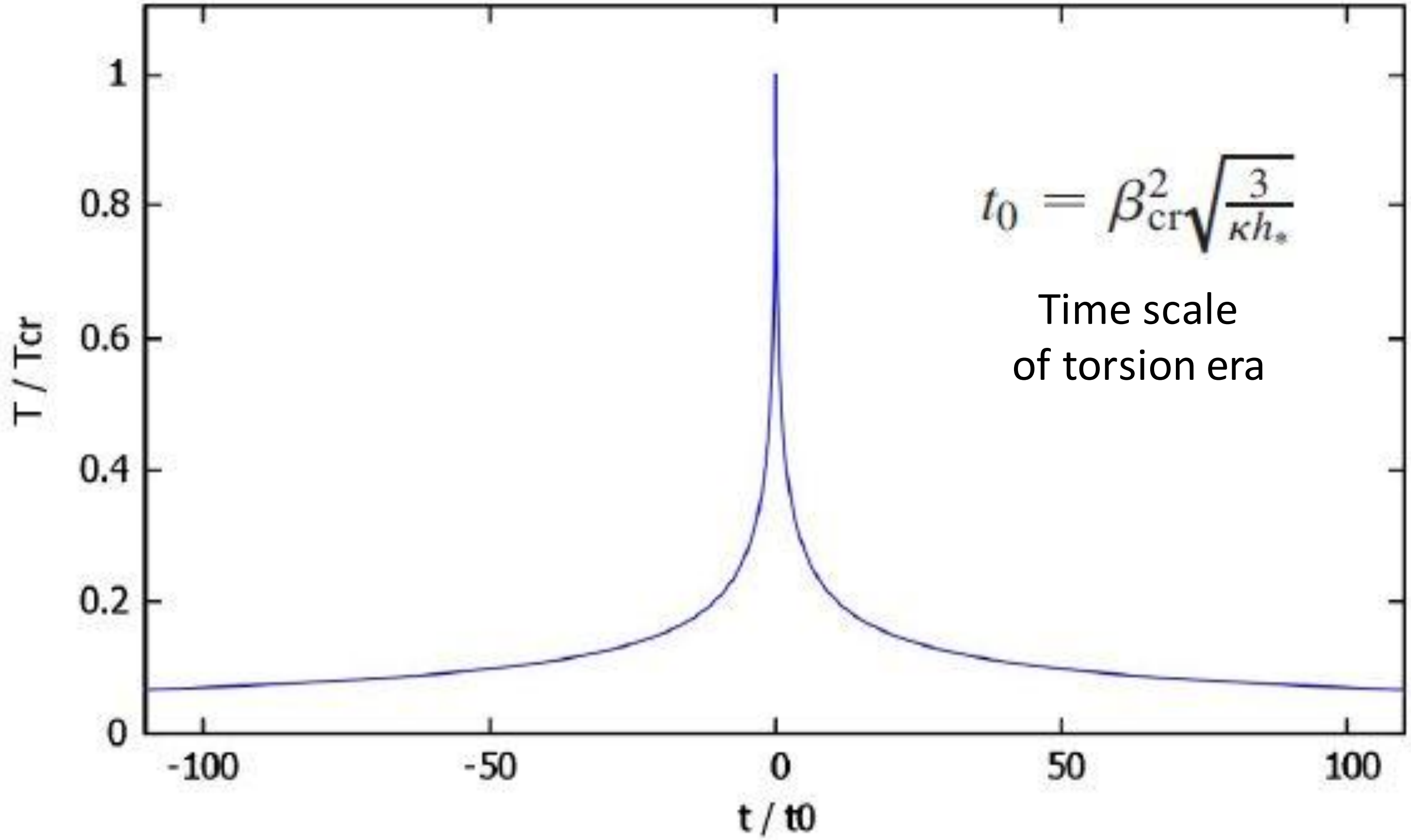
$$\beta = \sqrt{\frac{2}{3}}\beta_{cr} \cosh\eta \qquad \eta_{cr} = \operatorname{arcosh}\sqrt{\tfrac{3}{2}}.$$

$$\frac{t}{t_0} = \frac{1}{6}\sinh(2\eta) - \frac{2}{3}\eta + \frac{\sqrt{3}}{6} - \frac{2}{3}\eta_{cr}, \qquad \eta \leq -\eta_{cr}, \qquad t \leq 0$$

$$\frac{t}{t_0} = \frac{1}{6}\sinh(2\eta) - \frac{2}{3}\eta - \frac{\sqrt{3}}{6} + \frac{2}{3}\eta_{cr}, \qquad \eta \geq \eta_{cr}. \qquad t \geq 0$$

Cosmology with torsion

Temperature vs. time



$$t_0 = \beta_{\rm cr}^2 \sqrt{\frac{3}{\kappa h_*}}$$

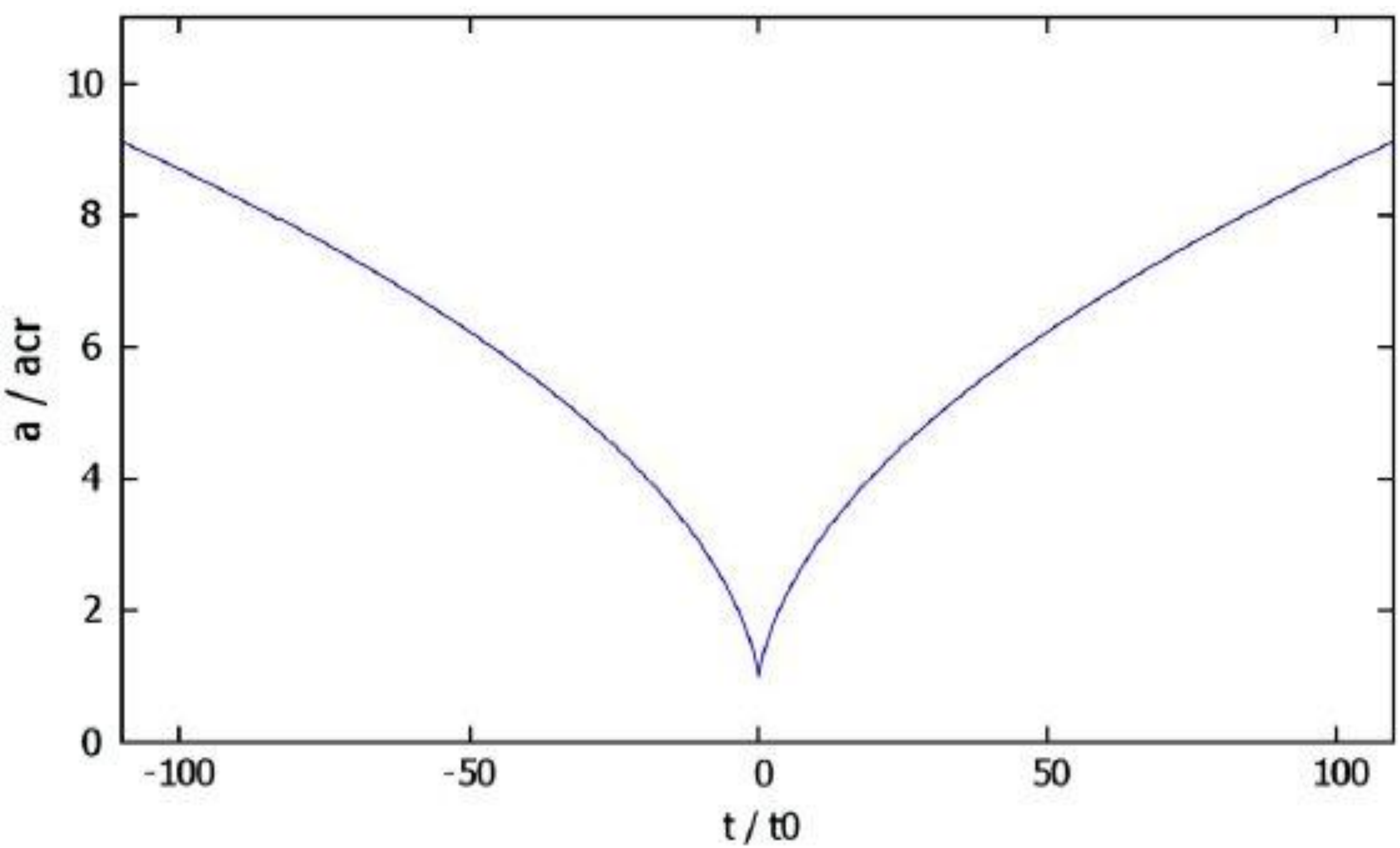
Time scale
of torsion era

Cusp-like bounce

Nonsingular big bounce instead of big bang

Scale factor vs. time

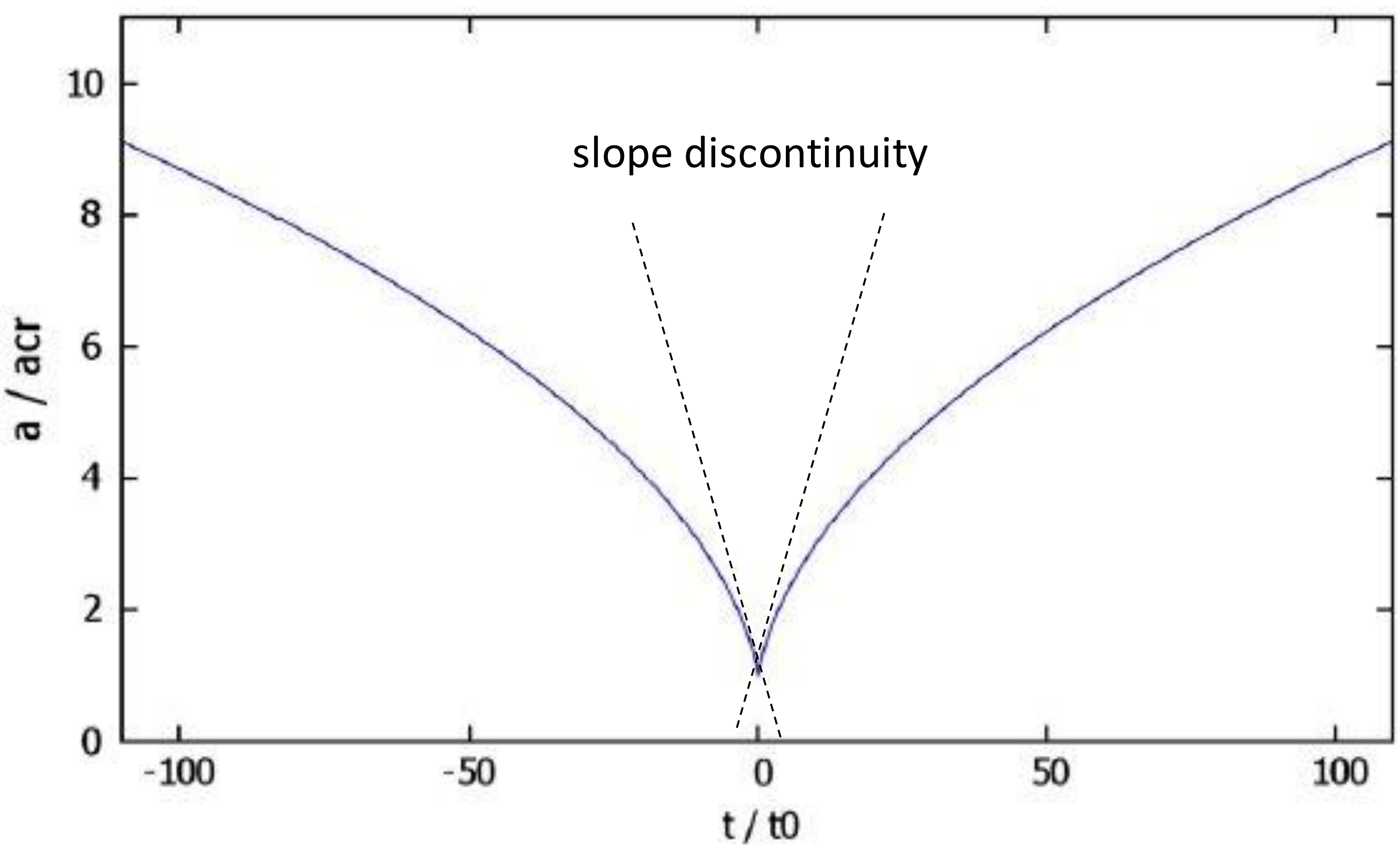
NP, Phys. Rev. D **85**, 107502 (2012)



Big bounce

Nonsingular big bounce instead of big bang

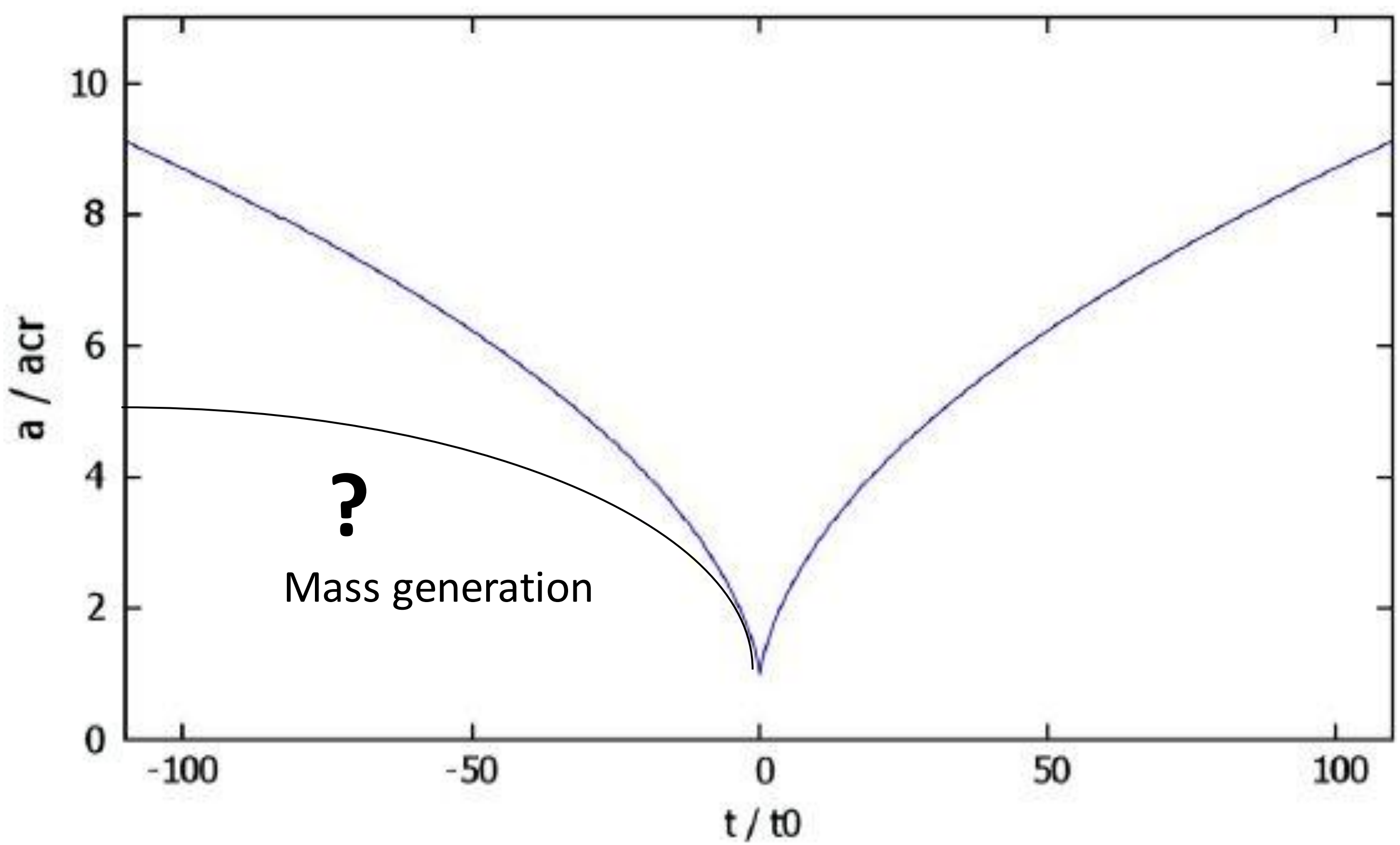
Scale factor vs. time



Big bounce

Nonsingular big bounce

Scale factor vs. time



Big bounce

Nonsingular big bounce

Singularity theorems?

Spinor-torsion coupling enhances strong energy condition

$$\tilde{\epsilon} + 3\tilde{p} = 2\alpha n^2 > 0$$

Expansion scalar (decreasing with time) in Raychaudhuri equation

$$\theta = \frac{3\dot{a}}{a}$$

is discontinuous at the bounce, preventing it from decreasing to $-\infty$ (reaching a singularity)

Torsion as alternative to inflation

For a closed Universe (k = 1):

Velocity of the antipode relative to the origin

$$v_{\mathrm{ant}}(T) = \pi\dot{a}(T)$$

At the bounce

$$|\dot{a}(T_{\mathrm{cr}})| = \left(\frac{32e}{243}\right)^{1/2}\frac{h_\star}{h_n}a_r T_r$$

Density parameter

$$\Omega(T) = 1 + \frac{1}{\dot{a}^2(T)}$$

Current values (WMAP)

$$\Omega = 1.002$$

$$a_0 = 2.9\times 10^{27}\ \mathrm{m}$$

NP, Phys. Rev. D **85**, 107502 (2012)

Torsion as alternative to inflation

Big bounce:

$$T_{\mathrm{cr}} \approx 0.78 m_{\mathrm{P}}$$

$$a_{\mathrm{cr}} \approx 5.9 \times 10^{-4}\ \mathrm{m}$$ ← Minimum scale factor

$$v_{\mathrm{ant}}(T_{\mathrm{cr}}) \approx 8.9 \times 10^{34}$$

$$N \sim v_{\mathrm{ant}}^{3}$$

↑

Number of causally disconnected volumes

Horizon problem solved

$$\Omega(T_{\mathrm{cr}}) \approx 1 + 1.3 \times 10^{-70}$$

Flatness problem solved

No free parameters

Cosmological perturbations – in progress

Theories of spacetime

General Relativity
Dynamical variables: matter fields + metric tensor

ECSK gravity
Dynamical variables: matter fields + metric tensor + torsion

Purely affine gravity
(A. Eddington 1922, A. Einstein 1923, E. Schrödinger 1950)
Dynamical variables: matter fields + **affine connection**

- Metric tensor is constructed from matter Lagrangian & curvature
- Field equations in vacuum generate cosmological constant
- Field equations with matter are more complicated and differ from (physical) metric solutions

Affine gravity

Similar to gauge theories of other fundamental forces:

- Affine connection (dynamical variable in affine gravity) generalizes an ordinary derivative to a coordinate-covariant derivative
- Gauge potentials (dynamical variables in gauge theories) generalize an ordinary derivative to gauge-invariant derivatives

Affine gravity

Dynamical Lagrangian must contain derivatives of connection

Simplest gravitational Lagrangian: linear in derivatives

→ linear in Ricci tensor (like in GR and ECSK) and contracted with an algebraic tensor constructed from connection (from torsion)

$$\boxed{k_{\mu\nu} = S^{\rho}{}_{\lambda\mu} S^{\lambda}{}_{\rho\nu}}$$

$$k^{\mu\rho} k_{\nu\rho} = \delta^{\mu}_{\nu} \qquad k = |\det(k_{\mu\nu})|$$

$$\det(k_{\mu\nu}) \neq 0$$

$$\boxed{\mathfrak{L}_{\mathrm{g}} = R_{\mu\nu} k^{\mu\nu} \sqrt{k}}$$

NP, ArXiv:1203.0294

Other Lagrangians based on

$$S^{\rho}{}_{\mu\nu} S_{\rho}, \quad m_{\mu\nu} = S_{\mu} S_{\nu}, \quad R^{\rho}{}_{\rho\mu\nu}$$

are unphysical

Affine gravity

Stationarity of action under $\delta\Gamma_{\mu\,\nu}^{\;\rho}$ → field equations

Variation can be split into $\delta\Gamma_{(\mu\,\nu)}^{\;\;\rho}$ and $\delta S^{\rho}{}_{\mu\nu}$

For vacuum:

$$\delta\Gamma_{(\mu\,\nu)}^{\;\;\rho} \rightarrow \; \Gamma_{\mu\,\nu}^{\;\rho} = \{{}_{\mu\,\nu}^{\;\rho}\}_k + S^{\rho}{}_{\mu\nu} - \frac{1}{3}(\delta^{\rho}_{\mu}S_{\nu} + \delta^{\rho}_{\nu}S_{\mu})$$

Christoffel symbols of tensor *k*

Gravitational Lagrangian becomes

$$\mathfrak{L}_{\mathrm{g}} = \left(R^{(k)}_{\mu\nu}k^{\mu\nu} + 4 - \frac{1}{3}m_{\mu\nu}k^{\mu\nu}\right)\sqrt{k}$$

Ricci tensor of tensor *k*

Cosmological-like term

Cosmological constant from torsion

Defining

$$\boxed{g_{\mu\nu} = \frac{2}{\Lambda} k_{\mu\nu}}$$

Λ **sets length scale of affine connection**

gives the Einstein-Hilbert action with cosmological constant

Affine length scale Λ becomes **cosmological constant**

Only configurations with $\det(k_{\mu\nu}) < 0$ are physical

c, *Λ*, *G* – fundamental constants of classical physics (set units)
Planck units set by *h*, *c*, *G* – their relation to *Λ* still unknown

NP, ArXiv:1203.0294

Cosmological constant from torsion

The metric in the matter Lagrangian must also be replaced by

$$g_{\mu\nu} = \frac{2}{\Lambda} k_{\mu\nu}$$

For ordinary matter (Dirac spinors, known gauge fields):
same gravitational Lagrangian

Total action

$$S = \int \left(R^{(k)}_{\mu\nu} k^{\mu\nu} + 4 - \frac{1}{3} m_{\mu\nu} k^{\mu\nu} \right) \sqrt{k}\, d\Omega + \underset{\uparrow}{\alpha} \int \mathfrak{L}_{\mathrm{m}}\, d\Omega$$

sets mass units

becomes the EH action with matter and Λ

$$G = -\frac{\alpha c^4}{8\pi\Lambda}$$

Cosmological constant from torsion

If fields depend on torsion only through $k_{\mu\nu}$ (spinors do not):

$$\delta S^{\rho}{}_{\mu\nu} \rightarrow \ S_{\mu} = 0,$$

$$R^{(k)}_{\mu\nu} - \frac{1}{2} R^{(k)}_{\rho\sigma} k^{\rho\sigma} k_{\mu\nu} = 2k_{\mu\nu} - \frac{\alpha}{\sqrt{k}} \frac{\delta \mathfrak{L}_{\mathrm{m}}}{\delta k^{\mu\nu}}$$

$$\Downarrow$$

$$R^{(g)}_{\mu\nu} - \frac{1}{2} R^{(g)}_{\rho\sigma} g^{\rho\sigma} g_{\mu\nu} = \Lambda g_{\mu\nu} - \frac{\alpha}{\Lambda} T_{\mu\nu}$$

Equations in the presence of spinors – in progress

Expected to reproduce or slightly modify ECSK with Λ

These modifications may contain $\Lambda^{1/2}c^2 \sim a_{\mathrm{MOND}}$ → dark matter

Summary

Torsion in the ECSK theory of gravity:

- Averts the big-bang singularity, replacing it by a nonsingular, cusp-like big bounce
- Solves the flatness and horizon problems without inflation

Torsion in the simplest affine theory of gravity:

- Gives field equations with a cosmological constant

No free parameters